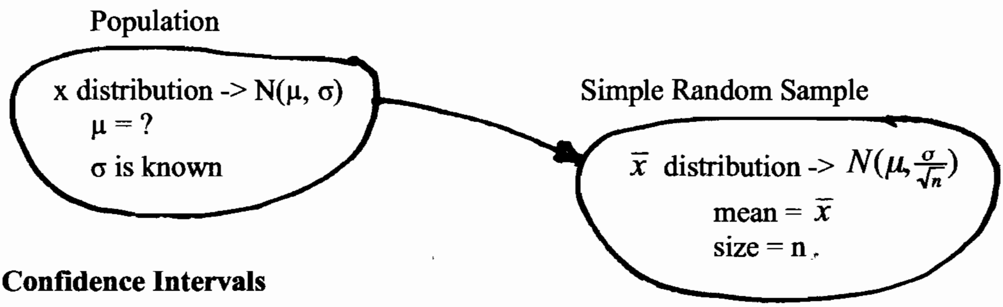
Situation 1. A population has a Normally distributed variable x. From this population we draw a Simple Random Sample of size n. The mean, μ, of the population is unknown. The standard deviation of the population, σ, is known. The mean of the sample, $\bar{x}$, is computed. The population variable x has the distribution $N(\mu, \sigma)$ and the sample mean $\bar{x}$ has the distribution $N(\mu, \frac{\sigma}{\sqrt{n}})$ Here is the picture:



Confidence Intervals

We can estimate μ with

$$\mu = \bar{x} \pm z^* \sigma / \sqrt{n}$$

Where z^* is determined by the confidence level C. The table below gives z^* for some common values of C:

C	90%	95%	99%
z^*	1.645	1.960	2.576

Hypothesis Testing.

The Null Hypothesis:
H_0: $\mu = \mu_0$

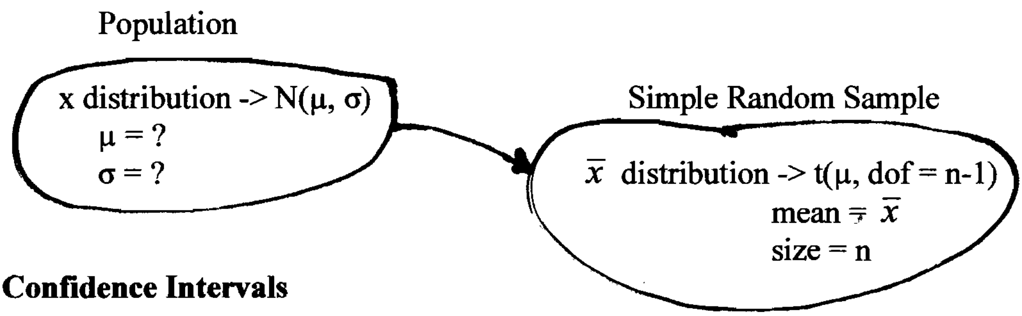
The Alternative Hypothesis:
H_a: $\mu > \mu_0$

First compute the z statistic using the formula:

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Then for significance level alpha determine z^*. Then if $z > z^*$ we reject the null hypothesis and say that we have statistically significant evidence for the alternative hypothesis. For the other alternative hypotheses, H_a: $\mu > \mu_0$ or H_a: $\mu \neq \mu_0$, we adjust the test appropriately.

Situation 2. A population has a Normally distributed variable x. From this population we draw a Simple Random Sample of size n. The mean, μ, of the population is unknown. **The standard deviation of the population, σ, is also unknown.** From the sample we compute both the mean $\bar{x}$ and the standard deviation s. The population variable x has the distribution $N(\mu, \sigma)$ and the sample mean $\bar{x}$ has the distribution t-distribution with mean μ and n-1 degrees of freedom. Here is the picture:



Confidence Intervals

We can estimate μ with

$$\mu = \bar{x} \pm t^{*} s / \sqrt{n}$$

Where t^* is determined by the confidence level C and the degrees of freedom of the t distribution (n-1). The table below is similar to the table for the z procedures:

C	90%	95%	99%
t^*	Depends on degrees of freedom		

Hypothesis Testing.

The Null Hypothesis:
H_0: $\mu = \mu_0$

The Alternative Hypothesis:
H_a: $\mu > \mu_0$

First compute the t statistic using the formula:

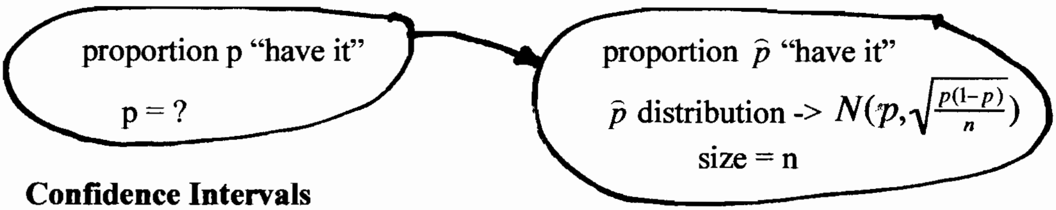
$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

Then for significance level alpha determine t^*. Then if $t > t^*$ we reject the null hypothesis and say that we have statistically significant evidence for the alternative hypothesis. For the other alternative hypotheses, H_a: $\mu > \mu_0$ or H_a: $\mu \neq \mu_0$, we adjust the test appropriately.

Situation 3. A proportion p of a population has some particular outcome of interest. From this population we draw a Simple Random Sample of size n. The proportion, p, of the population is unknown. The proportion of the sample with the outcome of interest is computed as $\hat{p} = \dfrac{\textit{number of successes in the sample}}{n}$. For a sufficiently large sample size the $\hat{p}$ statistic has the distribution $N(p, \sqrt{\frac{p(1-p)}{n}})$.

Here is the picture:

Population | Simple Random Sample



Confidence Intervals

We can estimate p with

$$p = \hat{p} \pm z^{*}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Where z^* is determined by the confidence level C. The table below gives z^* for some common values of C:

C	90%	95%	99%
z^*	1.645	1.960	2.576

Hypothesis Testing.

The Null Hypothesis:
H_0: $p = p_0$

The Alternative Hypothesis:
H_a: $p > p_0$

First compute the z statistic using the formula:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_o(1-p_0)}{n}}}$$

Then for significance level alpha determine z^*. Then if $z > z^*$ we reject the null hypothesis and say that we have statistically significant evidence for the alternative hypothesis. For the other alternative hypotheses, H_a: $p > p_0$ or H_a: $p \neq p_0$, we adjust the test appropriately.