

**Chapter 11 Formulas**

Let  $P=(x_0, y_0, z_0)$  and  $Q=(x_1, y_1, z_1)$  be points in 3-Space;  $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ ,  $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ ,  $\mathbf{w} = \langle w_1, w_2, w_3 \rangle$ ,  $\mathbf{n} = \langle a, b, c \rangle$ ,  $\mathbf{n}_1$ , and  $\mathbf{n}_2$  be vectors in 3-Space. Vectors are shown in bold for the questions. Use the arrow notation for vectors in your answers.

Give the formulas for the following:

<p>The dot product of <math>\mathbf{u}</math> and <math>\mathbf{v}</math>:</p> $\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$	<p>The cosine of the angle between <math>\mathbf{u}</math> and <math>\mathbf{v}</math>:</p> $\cos(\theta) = \frac{\vec{u} \cdot \vec{v}}{\ \vec{u}\  \ \vec{v}\ }$
<p><math>\mathbf{u}</math> and <math>\mathbf{v}</math> are orthogonal:</p> $\vec{u} \cdot \vec{v} = 0$	<p>The projection of <math>\mathbf{u}</math> onto <math>\mathbf{v}</math>:</p> $\text{proj}_{\vec{v}}(\vec{u}) = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{v}$
<p>The norm of <math>\mathbf{v}</math>:</p> $\ \vec{v}\  = \sqrt{\vec{v} \cdot \vec{v}}$ $= \sqrt{v_1^2 + v_2^2 + v_3^2}$	<p>The vector component of <math>\mathbf{u}</math> orthogonal to <math>\mathbf{v}</math>:</p> $\vec{u} - \text{proj}_{\vec{v}}(\vec{u})$ $= \vec{u} - \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{v}$
<p>The unit vector in the direction of <math>\mathbf{v}</math>:</p> $\frac{\vec{v}}{\ \vec{v}\ }$	<p>The cross product of <math>\mathbf{u}</math> and <math>\mathbf{v}</math>:</p> $\vec{u} \times \vec{v}$ $= (u_2 v_3 - u_3 v_2) \vec{i}$ $+ (u_3 v_1 - u_1 v_3) \vec{j}$ $+ (u_1 v_2 - u_2 v_1) \vec{k}$

<p>The vector equation of a line through P and parallel to <math>\langle a, b, c \rangle</math>:</p> $\vec{r}(t) = \langle x, y, z \rangle$ $= \langle x_0, y_0, z_0 \rangle + t \langle a, b, c \rangle$	<p>The standard equation of a plane through P with normal vector <math>\langle a, b, c \rangle</math>:</p> $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$
<p>The parametric equations of a line through P and parallel to v:</p> $x = x_0 + tv_1$ $y = y_0 + tv_2$ $z = z_0 + tv_3$	<p>The cosine of the angle between two planes with normal vectors <math>\vec{n}_1</math> and <math>\vec{n}_2</math>:</p> $\cos(\theta) = \frac{ \vec{n}_1 \cdot \vec{n}_2 }{\ \vec{n}_1\  \ \vec{n}_2\ }$
<p>The vector equation of a line through P and Q:</p> $\langle x, y, z \rangle$ $= (1-t) \langle x_0, y_0, z_0 \rangle + t \langle x_1, y_1, z_1 \rangle$	<p>The distance between the point Q and a plane through P with normal vector <math>\vec{n}</math>:</p> $\frac{ (\langle x_1, y_1, z_1 \rangle - \langle x_0, y_0, z_0 \rangle) \cdot \vec{n} }{\ \vec{n}\ }$
<p>The vector equation of a plane through P with normal vector <math>\vec{n} = \langle a, b, c \rangle</math>:</p> $(\langle x, y, z \rangle - \langle x_0, y_0, z_0 \rangle) \cdot \vec{n} = 0$	<p>The distance between the point Q and a line through P with direction vector <math>\vec{u}</math>:</p> $\frac{\ (\langle x_1, y_1, z_1 \rangle - \langle x_0, y_0, z_0 \rangle) \times \vec{u}\ }{\ \vec{u}\ }$
<p>The volume of the parallelepiped with vectors <math>\vec{u}</math>, <math>\vec{v}</math>, and <math>\vec{w}</math> as adjacent edges:</p> $\vec{u} \cdot (\vec{v} \times \vec{w})$ $= \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$	<p>The triple scalar product of <math>\vec{u}</math>, <math>\vec{v}</math>, and <math>\vec{w}</math>:</p> $\vec{u} \cdot (\vec{v} \times \vec{w})$ $= \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

also

$$\langle x, y, z \rangle =$$

$$\langle x_0, y_0, z_0 \rangle + t \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$$

## Chapter 12 Formulas

Let  $f(t)$  be a real valued function of  $t$ ;  $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$  and  $\mathbf{u}(t)$  be vector valued functions;  $\mathbf{r}(t)$  be the position vector,  $\mathbf{v}(t)$  be the velocity, and  $\mathbf{a}(t)$  be the acceleration;  $C$  be a smooth curve given by  $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$  on the interval  $(a, b)$ .

Give the formulas for the following:

$\mathbf{r}(t)$ is continuous at the point $t = a$ : $\lim_{t \rightarrow a} \vec{r}(t) = \vec{r}(a)$	$D_t [\mathbf{r}(f(t))] =$ $f'(t) \vec{r}'(f(t))$
The derivative of $\mathbf{r}(t)$ : $\vec{r}'(t) = \lim_{\Delta t \rightarrow 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}$	If $\mathbf{r}(t) \cdot \mathbf{r}(t) = \text{constant}$ then $\vec{r} \cdot \vec{r}' = 0$
$D_t [\mathbf{r}(t) + \mathbf{u}(t)] =$ $\vec{r}'(t) + \vec{u}'(t)$	$\int \vec{r}(t) dt =$ $\left( \int x(t) dt \right) \vec{i} + \left( \int y(t) dt \right) \vec{j} + \left( \int z(t) dt \right) \vec{k}$
$D_t [\mathbf{r}(t) \cdot \mathbf{u}(t)] =$ $\vec{r}' \cdot \vec{u} + \vec{r} \cdot \vec{u}'$	Velocity: $\mathbf{v}(t) =$ $\vec{r}'(t)$
$D_t [\mathbf{r}(t) \times \mathbf{u}(t)] =$ $\vec{r}' \times \vec{u} + \vec{r} \times \vec{u}'$	Acceleration: $\mathbf{a}(t) =$ $\vec{v}'(t) = \vec{r}''(t)$
$D_t [f(t)\mathbf{r}(t)] =$ $f'(t) \vec{r}(t) + f(t) \vec{r}'(t)$	Speed = $\  \vec{v}(t) \  = \  \vec{r}'(t) \ $

<p>Projectile position function for an initial velocity <math>\vec{v}_0</math> and an initial position <math>\vec{r}_0</math>: <math>\vec{r}(t) =</math></p> $\vec{r}(t) = -\frac{1}{2}gt^2\vec{j} + t\vec{v}_0 + \vec{r}_0$	<p>The arc length of C: <math>s =</math></p> $\int_a^b \ \vec{r}'(t)\  dt$ $= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt$
<p>The unit tangent vector: <math>\vec{T}(t) =</math></p> $\frac{\vec{r}'(t)}{\ \vec{r}'(t)\ }$	<p>The arc length function on C: <math>s(t) =</math></p> $\int_a^t \ \vec{r}'(u)\  du$
<p>Principle unit normal vector: <math>\vec{N}(t) =</math></p> $\frac{\vec{T}'(t)}{\ \vec{T}'(t)\ }$	<p>The curvature for C given by the arc length parameterization <math>\vec{r}(s)</math>: <math>K =</math></p> $\ \vec{T}'(s)\  = \ \vec{r}''(s)\ $
<p>Acceleration as a linear combination of <math>\vec{T}</math> and <math>\vec{N}</math>:</p> $\vec{a} = a_T \vec{T} + a_N \vec{N}$	<p>The curvature for C given by <math>\vec{r}(t)</math>: <math>K =</math></p> $\frac{\ \vec{T}'(t)\ }{\ \vec{r}'(t)\ } = \frac{\ \vec{r}' \times \vec{r}''\ }{\ \vec{r}'\ ^3}$
<p>The tangential component of acceleration: <math>a_T =</math></p> $D_t \ \vec{v}(t)\  = \frac{\vec{r}' \cdot \vec{r}''}{\ \vec{r}'\ }$ $= \frac{d^2s}{dt^2}$	<p>Acceleration in terms of speed <math>(ds/dt)</math> and curvature:</p> $\vec{a}(t) = \frac{d^2s}{dt^2} \vec{T} + K \left(\frac{ds}{dt}\right)^2 \vec{N}$
<p>The normal or centripetal component of acceleration: <math>a_N =</math></p> $\sqrt{\ \vec{r}''\ ^2 - a_T^2}$ $= \frac{\ \vec{r}' \times \vec{r}''\ }{\ \vec{r}'\ } = \ \vec{a} \times \vec{T}\ $	<p>A vector orthogonal to the unit vector <math>x(t)\vec{i} + y(t)\vec{j}</math>:</p> $-y(t)\vec{i} + x(t)\vec{j}$ <p>or</p> $y(t)\vec{i} - x(t)\vec{j}$