1. Find and simplify the function values.

$f(x,y) = 5 - x^2 - 10y^2$

(i) $f(0,0)$ (ii) $f(0,1)$ (iii) $f(3,9)$

(iv) $f(1,y)$ (v) $f(x,0)$ (vi) $f(t,1)$

2. Find and simplify the function values.

$h(x,y,z) = \dfrac{xy}{z}$

(i) $h(7,8,24)$ (ii) $h(6,5,6)$ (iii) $h(-7,8,9)$ (iv) $h(10,9,-16)$

3. Find and simplify the function values.

$g(x,y) = \int_x^y (16t - 6)dt$

(i) $g(0,32)$ (ii) g(1,32) (iii) $g\left(\frac{3}{8},32\right)$ (iv) $g\left(0,\frac{3}{8}\right)$

4. Describe the domain and range of the function.

$f(x,y) = \sqrt{100 - x^2 - y^2}$

5. Describe the level curves of the function. Sketch the level curves for the given *c*-values.

   $z = 6 - 2x - 3y$, $c = 0, 2, 4, 6$

6. Find the limit and discusss the continuity of the function.

   $$\lim_{(x,y)\to(-5,4)} \left(x + 7y^2\right)$$

7. Find the limit and discusss the continuity of the function.

   $$\lim_{(x,y)\to(2,-10)} \left(8x + y + 2\right)$$

8. Find the limit and discusss the continuity of the function.

   $$\lim_{(x,y)\to(1,1)} \frac{x}{\sqrt{7x+6y}}$$

9. Find the limit and discusss the continuity of the function.

   $$\lim_{(x,y,z)\to(-1,0,8)} 5xe^{y^6 z}$$

10. Find the limit (if it exists). If the limit does not exist, explain why.

$$\lim_{(x,y)\to(4,3)} \frac{xy-4}{3+xy}$$

11. Discusss the continuity of the function at the origin.

$$f(x,y)=\begin{cases} \dfrac{4x^8y^8}{x^8+y^8}, & (x,y)\neq(0,0) \\ 2, & (x,y)=(0,0) \end{cases}$$

12. Use polar coordinates to find the limit. [Hint: Let $x = r\cos\theta$ and $y = r\sin\theta$, and note that $(x,y)\to(0,0)$ implies $r\to 0$.]

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2}$$

13. Discusss the continuity of the function.

$$f(x,y,z)=\frac{z}{x^2+y^2-64}$$

14. Find each limit for the function $f(x,y) = -7x^2 - 4y$.

(i) $\lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$

(ii) $\lim\limits_{\Delta y \to 0} \dfrac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$

15. Find both first partial derivatives.

$f(x,y) = -4x + 3y - 8$

16. Find both first partial derivatives.

$f(x,y) = -4x^2 + 5y^2 + 1$

17. Find both first partial derivatives.

$z = x \cdot \sqrt[4]{y}$

18. Find both first partial derivatives.

$z = 2y^3\sqrt{x}$

19. Find both first partial derivatives.

$$z = x^6 e^{10y}$$

20. Find both first partial derivatives.

$$f(x, y) = \ln\left(x^5 + y^8\right)$$

21. Find both first partial derivatives.

$$f(x, y) = \ln \sqrt[7]{xy}$$

22. Find both first partial derivatives.

$$z = \frac{x^2}{9y} + \frac{5y^2}{x}$$

23. Find both first partial derivatives.

$$f(x, y) = \sqrt{4x^{10} + y^4}$$

24. Evaluate f_x and f_y at the given point.

$$f(x, y) = \frac{3xy}{\sqrt{9x^2 + 2y^2}}, \quad (1,1)$$

25. For $f(x,y)$, find all values of x and y such that $f_x(x,y) = 0$ and $f_y(x,y) = 0$ simultaneously.

$$f(x, y) = 9x^3 - 3xy + 9y^3$$

26. Find the first partial derivatives with respect to x, y, and z.

$$w = \frac{2xz}{9x + 6y}$$

27. Find the first partial derivatives with respect to x, y, and z.

$$H(x, y, z) = \cos(2x + 8y + 7z)$$

28. Evaluate f_x and f_y at the given point.

$$f(x, y, z) = \frac{xy}{x + y + z}, \quad (6,8,3)$$

29. Find the four second partial derivatives. Observe that the second mixed partials are equal.

$$z = x^2 + 2xy + 8y^2$$

30. Find the four second partial derivatives. Observe that the second mixed partials are equal.

$$z = 11xe^y + 8ye^{-x}$$

31. Find the total differential of the function $z = 5x^{10}y^9$.

32. Find the total differential of the function $z = \dfrac{x^9}{y}$.

33. Find the total differential of the function $z = -\dfrac{1}{x^7 + y^5}$.

34. For the function $f(x, y) = 5x - 4y$:

(i) Evaluate *f*(1,5) and *f*(1.09,5.09) and calculate Δz, and

(ii) Use the total differential *dz* to approximate Δz.

35. For the function $f(x, y) = x \sin y$:

(i) Evaluate $f(4,2)$ and $f(4.08,2.03)$ and calculate Δz, and

(ii) Use the total differential dz to approximate Δz.

36. The radius r and height h of a right circular cylinder are measured with possible errors of 8% and 3%, respectively. Approximate the maximum possible percent error in measuring the volume.

37. A triangle is measured and two adjacent sides are found to be 3 inches and 4 inches long, with an included angle of $\frac{\pi}{4}$. The possible errors in measurement are $\frac{1}{10}$ inch for the sides and 0.04 radian for the angle. Approximate the maximum possible error in the computation of the area.

38. Let $w = xy$, where $x = 10\sin t$ and $y = -8\cos t$. Find $\frac{dw}{dt}$.

39. Let $w = \cos(2x - 4y)$, where $x = t^9$ and $y = 7$. Find $\frac{dw}{dt}$.

40. Let $w = xy\cos z$, where $x = t^9$, $y = t^2$, and $z = \arccos t$. Find $\dfrac{dw}{dt}$.

41. Let $w = x^3 + y^3$, where $x = 4s + t$, $y = 4s - t$. Find $\dfrac{\partial w}{ds}$ and $\dfrac{\partial w}{dt}$ and evaluate each partial derivative at the point $s = 2$, $t = 2$.

42. Let $w = x^7 - 7x^6 y$, where $x = e^s$, $y = e^t$. Find $\dfrac{\partial w}{ds}$ and $\dfrac{\partial w}{dt}$ and evaluate each partial derivative at the point $s = 0$, $t = 1$.

43. Let $w = (x - y)^3$, where $x = r + \theta$ and $y = r - \theta$. Find $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial \theta}$.

44. Let $w = \dfrac{yz}{x}$, where $x = \theta^2$, $y = 9r + 7\theta$, and $z = 9r - 7\theta$. Find $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial \theta}$.

45. Differentiate implicitly to find $\dfrac{dy}{dx}$.

$$x^2 - 9xy + y^2 - 6x + y - 7 = 0$$

46. Differentiate implicitly to find the first partial derivatives of z.

$$x^9 + y^9 + z^9 = 8$$

47. Differentiate implicitly to find the first partial derivatives of z.

$$x^3 + \sin(6y + 7z) = 0$$

48. Differentiate implicitly to find the first partial derivatives of w.

$$x^6 + y^6 + z^6 - 7yw + 9w^{10} = 10$$

49. The radius of a right cylinder is increasing at a rate of 2 inches per minute, and the height is decreasing at a rate of 3 inches per minute. What is the rate of change of the volume and surface area (including both ends of the cylinder as well as the side) when the radius is 9 inches and height is 27 inches?

50. Find the directional derivative of the function at P in the direction of $\vec{\mathbf{v}}$.

$$f(x, y) = 5x - 7xy + 10y, \quad P(1, 4), \quad \vec{\mathbf{v}} = \frac{1}{2}\left(\hat{\mathbf{i}} + \sqrt{3}\hat{\mathbf{j}}\right)$$

51. Find the directional derivative of the function at P in the direction of $\vec{\mathbf{v}}$.

$$f(x, y) = x^3 - y^3, \quad P(2,1), \quad \vec{\mathbf{v}} = \frac{\sqrt{2}}{2}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$$

52. Find the directional derivative of the function at P in the direction of $\vec{\mathbf{v}}$.

$$f(x, y, z) = xy + yz + xz, \quad P(1,1,1), \quad \vec{\mathbf{v}} = 9\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 10\hat{\mathbf{k}}$$

53. Find the directional derivative of the function in the direction of $\vec{\mathbf{u}} = \cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}$.

$$f(x, y) = \sin(3x - 2y), \quad \theta = \frac{\pi}{3}$$

54. Find the gradient of the function at the given point.

$$f(x, y) = 9x - 4y^2 + 2, \quad (4,1)$$

55. Find the gradient of the function at the given point.

$$g(x, y) = 9xe^{\frac{y}{x}}, \quad (9,0)$$

56. Find the gradient of the function at the given point.

$w = 6x^2y - 10yz + z^2, \quad (1,1,-2)$

57. Use the gradient to find the directional derivative of the function at P in the direction of Q.

$g(x,y) = x^2 + y^2 + 1, \quad P(2,4) \quad Q(6,12)$

58. Use the gradient to find the directional derivative of the function at P in the direction of Q.

$f(x,y) = \sin(7x)\cos y, \quad P(0,0), \quad Q(\frac{\pi}{7}, \pi)$

59. Find the gradient of the function and the maximum value of the directional derivative at the given point.

$w = xy^4z^7, \quad (9,1,1)$

60. Find the directional derivative $D_{\mathrm{u}} f(3,2)$ of the function $f(x,y) = 10 - \frac{x}{10} - \frac{y}{3}$ in the direction of $\vec{\mathbf{u}} = \cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}$.

(i) $\theta = \frac{\pi}{4}$; (ii) $\theta = \frac{2\pi}{3}$

61. Find the directional derivative $D_{\mathbf{u}} f(3,2)$ of the function $f(x,y) = 10 - \frac{x}{10} - \frac{y}{5}$ in the direction of $\vec{\mathbf{u}} = \frac{\vec{\mathbf{v}}}{\|\vec{\mathbf{v}}\|}$.

(i) $\vec{\mathbf{v}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$; (ii) $\vec{\mathbf{v}} = -3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$

62. Find the directional derivative $D_{\mathbf{u}} f(3,2)$ of the function $f(x,y) = 10 - \frac{x}{10} - \frac{y}{5}$ in the direction of $\vec{\mathbf{u}} = \frac{\vec{\mathbf{v}}}{\|\vec{\mathbf{v}}\|}$.

(i) $\vec{\mathbf{v}}$ is the vector from (1,2) to (–2 ,6); (ii) $\vec{\mathbf{v}}$ is the vector from (3,2) to (4,5).

63. Find $\nabla f(x,y)$ for function $f(x,y) = 5 - \frac{x}{5} - \frac{y}{8}$.

64. For function $f(x,y) = 6 - \frac{x}{6} - \frac{y}{9}$, find the maximum value of the directional derivative at (3,2).

65. Use the gradient to find a normal vector to the graph of the equation at the given point.

$9x^2 - y = 4, \quad (10, 896)$

66. Find a unit normal vector to the surface $x + y + z = 6$ at the point (3,0,3).

67. Find a unit normal vector to the surface $x^2 + y^2 + z^2 = 18$ at the point (4,1,1).

68. Find a unit normal vector to the surface $x^4 y^2 - z = 0$ at the point $(1, 6, 36)$.

69. Find a unit normal vector to the surface $x^2 + 2y + z^3 = 10$ at the point $(2, -1, 2)$.

70. Find an equation of the tangent plane to the surface $g(x, y) = x^2 - y^2$ at the point $(4, 6, -20)$.

71. Find an equation of the tangent plane to the surface $f(x, y) = 10 - \frac{5}{4}x - y$ at the point $(8, -1, 1)$.

72. Find an equation of the tangent plane to the surface $x^2 + 9y^2 + z^2 = 504$ at the point $(6, -6, 12)$.

73. Find an equation of the tangent plane to the surface $x = y(7z - 5)$ at the point $(96, 6, 3)$.

74. Find an equation of the tangent plane and find symmetric equations of the normal line to the surface $x^2 + y^2 + z^2 = 201$ at the point $(1, 10, 10)$.

75. Find an equation of the tangent plane and find symmetric equations of the normal line to the surface $xy - 4z = 0$ at the point $(-4, -10, 10)$.

76. Find an equation of the tangent plane and find symmetric equations of the normal line to the surface $xyz = 30$ at the point $(1, 3, 10)$.

77. Find the angle of inclination θ of the tangent plane to the surface $x^2 - y^2 + z = 0$ at the point $(1, 3, 8)$.

78. Find the angle of inclination θ of the tangent plane to the surface $3x^2 + 4y^2 - z = 0$ at the point $(6, 6, 252)$.

79. Find the distance from the point (0,0,0) to the plane $9x + 10y + z = 5$.

80. Find three positive numbers *x*, *y*, and *z* whose sum is 3 and product is a maximum.

81. Find three positive numbers *x*, *y*, and *z* whose sum is 24 and the sum of the squares is a maximum.

82. The sum of the length (denote by *z*) and the girth (perimeter of a cross section) of packages carried by a delivery service cannot exceed 36 inches. Find the dimensions of the rectangular package of largest volume that may be sent.

83. The material for constructing the base of an open box costs 1.5 times as much per unit area as the material for constructing the sides. For a fixed amount of money 250.00, find the dimensions of the box of largest volume that can be made.

84. A company manufactures two types of sneakers: running shoes and basketball shoes. The total revenue from x_1 units of running shoes and y_1 units of basketball shoes is:

$$R = -3x_1^2 - 8x_2^2 - 2x_1x_2 + 40x_1 + 109x_2 \ ,$$

where x_1 and x_2 are in thousands of units. Find x_1 and x_2 so as to maximize the revenue.

85. Find the least squares regression line for the points (1,0) , (3,3) , (10,6).

86. A store manager wants to know the demand *y* for an energy bar as a function of price *x*. The daily sales for three different prices of the energy bar are shown in the table.

Price, *x*	\$ 1.06	\$ 1.21	\$ 1.45
Demand, *y*	420	375	390

(i) Use the regression capabilities of a graphing utility to find the least squares regression line for the data.

(ii) Use the model to estimate the demand when the price is 1.33.