M252 Practice Exam 3 Computation Section

1. Find the domain of the vector-valued function given below.

   $$\mathbf{r}(t)=\sqrt{16-t^2}\,\mathbf{i}+\frac{1}{t+6}\mathbf{j}+(t-4)\mathbf{k}$$

   A) $-\infty < t < \infty$

   B) $(\infty,\ 4]$

   C) $[0,\ 4]$

   D) $[-4, 4]$

   E) $[-4, \infty)$

2. Find the domain of the vector-valued function given below.

   $$\mathbf{r}(t)=\mathbf{F}(t)\times\mathbf{G}(t)$$

   where

   $$\mathbf{F}(t)=t^3\mathbf{i}+\frac{1}{t+6}\mathbf{j}+(t-6)\mathbf{k}$$

   $$\mathbf{G}(t)=\sqrt{9-t^2}\,\mathbf{i}+t\mathbf{j}+(t-3)\mathbf{k}$$

   A) $[-3,3]$

   B) $[3,\infty)$

   C) $[0,3]$

   D) $-\infty \le t \le \infty$

   E) $[-6,\infty)$

3. Evaluate (if possible) the vector-valued function at the point given.

   $$\mathbf{r}(t)=4t^5\mathbf{i}+\frac{1}{t+4}\mathbf{j},\quad \mathbf{r}(3)$$

   A) $138.857143\mathbf{i}+0.14\mathbf{j}$

   B) $972\mathbf{j}+0.14\mathbf{i}$

   C) $972\mathbf{i}+0.14\mathbf{j}$

   D) $0.14\mathbf{i}+972\mathbf{j}$

   E) $972\mathbf{i}+138.857143\mathbf{j}$

4. Given the vector-valued function below, evaluate $\mathbf{r}(1+\Delta t)-\mathbf{r}(1)$.

$\mathbf{r}(t)=\ln t\ \mathbf{i}+\frac{1}{t}\ \mathbf{j}+5t\,\mathbf{k},$

A) $\ln\left(1+\frac{\Delta t}{1}\right)\mathbf{i}+\frac{1\Delta t}{(1+\Delta t)t}\mathbf{j}+5\Delta t\,\mathbf{k}$

B) $\ln\left(1+\frac{\Delta t}{1}\right)\mathbf{i}-\frac{1\Delta t}{(1+\Delta t)t}\mathbf{j}+5\Delta t\,\mathbf{k}$

C) $\ln\left(1-\frac{\Delta t}{1}\right)\mathbf{i}-\frac{1\Delta t}{(1+\Delta t)t}\mathbf{j}+5\Delta t\,\mathbf{k}$

D) $\ln\left(1+\frac{\Delta t}{1}\right)\mathbf{i}-\frac{1\Delta t}{(1+\Delta t)}\mathbf{j}+5\Delta t\,\mathbf{k}$

E) $\ln\left(1+\frac{\Delta t}{1}\right)\mathbf{i}-\frac{1\Delta t}{(1+\Delta t)t}\mathbf{j}+5\ \mathbf{k}$

5. Find $\|\mathbf{r}(t)\|$ given the function $\mathbf{r}(t)$ below.

$\mathbf{r}(t)=4\sin 2t\ \mathbf{i}+4\cos 2t\ \mathbf{j}+6t\,\mathbf{k}$

A) $\sqrt{16+6t}$

B) $16+36t^2$

C) $4+6t$

D) $\sqrt{64+36t^2}$

E) $\sqrt{16+36t^2}$

6. Find $\|\mathbf{r}(t)\|$ given the function $\mathbf{r}(t)$ below.

$\mathbf{r}(t)=2t^{3/5}\,\mathbf{i}+3t\ \mathbf{j}+6t^2\,\mathbf{k}$

A) $\sqrt{2t^{6/5}+9t^2+36t^4}$

B) $\sqrt{4t^{3/5}+9t^2+36t^4}$

C) $\sqrt{4t^{6/5}+9t+36t^4}$

D) $\sqrt{4t^{6/5}+9t^2+36t^4}$

E) $\sqrt{2t^{6/5}+3t^2+6t^4}$

7. Find **r**(t).**u**(t) given the functions below.

$$\mathbf{r}(t) = 5t\ \mathbf{i} + \frac{t}{7}\ \mathbf{j} + 5t^2\ \mathbf{k}, \qquad \mathbf{u}(t) = 8t\ \mathbf{i} + 8\ \mathbf{j} - 7t\,\mathbf{k}$$

A) $40t + \frac{8}{7}t - 35t^2$

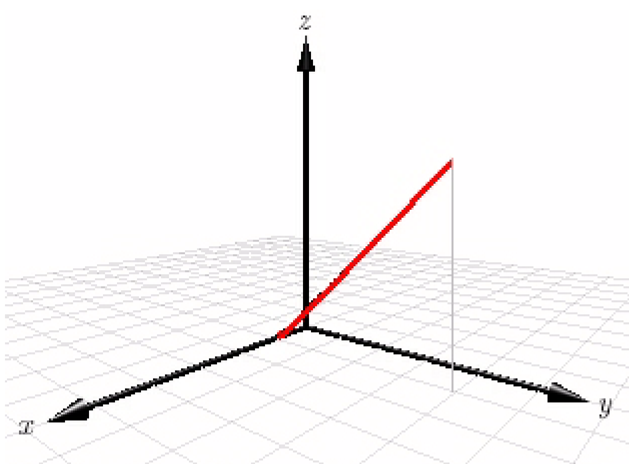
B) $40t^2 + \frac{8}{7}t + 35t^3$

C) $35t^2 + \frac{8}{7}t + 40t^3$

D) $40t^2 + \frac{8}{7}t - 35t^3$

E) $40t^2 + \frac{8}{7} - 35t^3$

8. Match the equation with the graph shown in red below.



A) $\mathbf{r}(t) = \mathbf{i} + \mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 1$

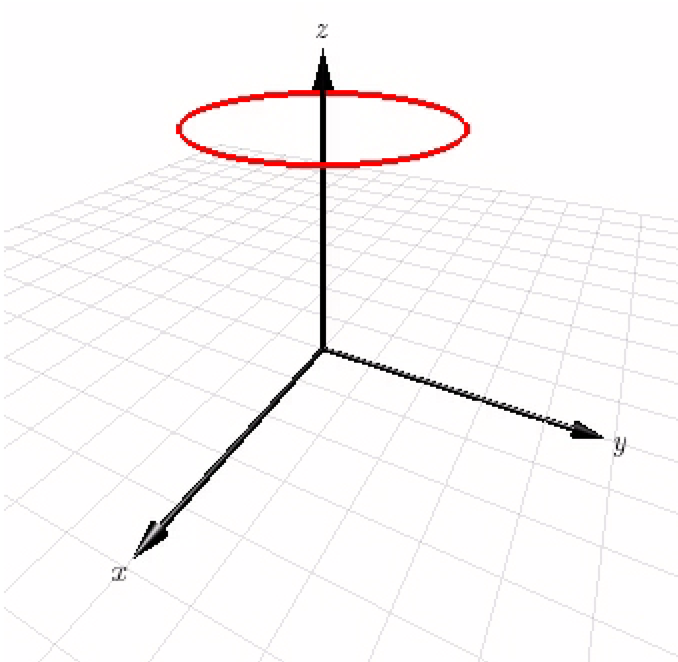
B) $\mathbf{r}(t) = t\mathbf{i} + t\,\mathbf{j} + t^2\mathbf{k}, \quad 0 \le t \le 1$

C) $\mathbf{r}(t) = \mathbf{i} + t\,\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 4$

D) $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 1$

E) None of the above

9. Match the equation with the graph shown in red below.



A) $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + 2\mathbf{k}, \quad 0 \le t \le 2\pi$

B) $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + \frac{t}{10}\mathbf{k}, \quad 0 \le t \le 4\pi$

C) $\mathbf{r}(t) = 2\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j} + 2\mathbf{k}, \quad 0 \le t \le 2\pi$

D) $\mathbf{r}(t) = 2\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j} + \mathbf{k}, \quad 0 \le t \le 4\pi$

E) None of the above

10. Represent the following curve by a vector valued function.

$$\frac{x^2}{9}+\frac{y^2}{36}=1,\ x>0$$

A) $\mathbf{r}(t)=3\cos 2\pi t\ \mathbf{i}+6\sin 2\pi t\ \mathbf{j},\ -\frac{1}{4}\le t\le\frac{1}{4}$

B) $\mathbf{r}(t)=3\cos \pi t\ \mathbf{i}-6\sin \pi t\ \mathbf{j},\ -\frac{1}{2}\le t\le\frac{1}{2}$

C) $\mathbf{r}(t)=3\cos t\ \mathbf{i}+6\sin t\ \mathbf{j},\ -\frac{\pi}{2}\le t\le\frac{\pi}{2}$

D) $\mathbf{r}(t)=\frac{3}{6}\sqrt{36-t^2}\ \mathbf{i}+t\ \mathbf{j},\ -6\le t\le 6$

E) All of the above

11. Find a vector-valued function, using the given parameter, to represent the intersection of the surfaces given below.

Surfaces	Parameter
$z=\frac{x^2}{49}+\frac{y^2}{16},\ y+9x=0$	$x=t$

A) $\mathbf{r}(t)=t\ \mathbf{i}+9t\mathbf{j}+\frac{3985}{784}t^2\,\mathbf{k}$

B) $\mathbf{r}(t)=t\ \mathbf{i}-9t\mathbf{j}+\frac{3985}{784}t^2\,\mathbf{k}$

C) $\mathbf{r}(t)=t\ \mathbf{i}-9t\mathbf{j}+\frac{784}{3985}t^2\,\mathbf{k}$

D) $\mathbf{r}(t)=t\ \mathbf{i}-9t\ \mathbf{k}+\frac{3985}{784}t^2\,\mathbf{j}$

E) None of the above

12. Find a vector-valued function, using the given parameter, to represent the intersection of the surfaces given below.

Surfaces	Parameter
$z = x^2 + y^2$, $z = 25$	$x = 5\cos 2\pi t$

A) $\mathbf{r}(t) = 5\cos 2\pi t\ \mathbf{i} + 5\sin \pi t\mathbf{j} + 25\mathbf{k}$
B) $\mathbf{r}(t) = 5\cos 2\pi t\ \mathbf{i} + 5\sin 2\pi t\mathbf{k} + 25\mathbf{j}$
C) $\mathbf{r}(t) = 5\cos 2\pi t\ \mathbf{i} + 5\sin 2\pi t\mathbf{j} + 5\mathbf{k}$
D) $\mathbf{r}(t) = 5\cos 2\pi t\ \mathbf{i} + 5\sin 2\pi t\mathbf{j} + 25\mathbf{k}$
E) None of the above

13. Find a vector-valued function, using the given parameter, to represent the intersection of the surfaces given below.

Surfaces	Parameter
$x^2 + y^2 = 81$, $z = x^2$	$x = 9\sin 4\pi t$

A) $\mathbf{r}(t) = 81\sin 4\pi t\ \mathbf{i} + 81\cos 4\pi t\mathbf{j} + 9\sin^2 4\pi t\mathbf{k}$
B) $\mathbf{r}(t) = 9\sin 4\pi t\ \mathbf{i} + 9\cos 4\pi t\mathbf{j} - 81\sin^2 4\pi t\mathbf{k}$
C) $\mathbf{r}(t) = 9\sin 4\pi t\ \mathbf{i} + 9\cos 4\pi t\mathbf{j} + 81\sin^2 4\pi t\mathbf{k}$
D) $\mathbf{r}(t) = 9\sin 4\pi t\ \mathbf{j} + 9\cos 4\pi t\mathbf{i} + 81\sin^2 4\pi t\mathbf{k}$
E) None of the above

14. Find a vector-valued function, using the given parameter, to represent the intersection of the surfaces given below.

Surfaces	Parameter
$x^2 + y^2 + z^2 = 68$, $x + y = 6$	$x = 3 + 5\sin t$

A) $\mathbf{r}(t) = (3+5\sin t)\ \mathbf{i} + (3-5\cos t)\mathbf{j} + \sqrt{25}\cos t\ \mathbf{k}$
B) $\mathbf{r}(t) = (3+5\sin t)\ \mathbf{i} + (3-5\sin t)\mathbf{j} + 5\sqrt{2}\cos t\ \mathbf{k}$
C) $\mathbf{r}(t) = (3+5\sin t)\ \mathbf{i} + (3-5\sin t)\mathbf{j} + 5\sqrt{2}\sin t\ \mathbf{k}$
D) $\mathbf{r}(t) = (3+5\sin t)\ \mathbf{i} + (3-5\cos t)\mathbf{j} + 5\sqrt{2}\cos t\ \mathbf{k}$
E) None of the above

15. Evaluate the limit given below.

$$\lim_{t\to 8}\left(3t\,\mathbf{i}+\frac{t^2-64}{t^2-8t}\,\mathbf{j}+\frac{17}{t}\,\mathbf{k}\right)$$

A) $24\,\mathbf{i}+\frac{17}{8}\,\mathbf{k}$

B) $24\,\mathbf{i}+\mathbf{j}+\frac{17}{8}\,\mathbf{k}$

C) $3\,\mathbf{i}+64\,\mathbf{j}+\frac{17}{8}\,\mathbf{k}$

D) $24\,\mathbf{i}+2\mathbf{j}+\frac{17}{8}\,\mathbf{k}$

E) The limit does not exist.

16. Evaluate the limit given below.

$$\lim_{t\to \infty}\left(e^{-3t}\,\mathbf{i}+\frac{2t^4}{t^4+6t}\,\mathbf{j}+\frac{4}{t}\,\mathbf{k}\right)$$

A) $\mathbf{r}(t)=\mathbf{0}$

B) $\mathbf{r}(t)=\mathbf{i}$

C) $\mathbf{r}(t)=\mathbf{i}+2\mathbf{j}$

D) $\mathbf{r}(t)=2\,\mathbf{j}$

E) The limit does not exist.

17. On a sketch of the plane curve represented by the vector valued function

$$\mathbf{r}(t) = \left(5+5t\right)\,\mathbf{i} + \left(2+3t^2\right)\,\mathbf{j},$$

sketch the vectors $\mathbf{r}'(3)$ and $\mathbf{r}'(3)$. Position the vectors so that the initial point of $\mathbf{r}(3)$ is at the origin and the initial point of $\mathbf{r}'(3)$ is at the terminal point of $\mathbf{r}(3)$.

What is the relationship between $\mathbf{r}'(3)$ and the curve?

	$\mathbf{r}(3)$	$\mathbf{r}'(3)$	Relationship
A)	$\mathbf{r}(3) = 20\,\mathbf{i} + 29\,\mathbf{j}$	$\mathbf{r}'(3) = 10\,\mathbf{i} + 18\,\mathbf{j}$	$\mathbf{r}'(3)$ is tangent to the curve
B)	$\mathbf{r}(3) = 20\,\mathbf{i} + 29\,\mathbf{j}$	$\mathbf{r}'(3) = 5\,\mathbf{i} + 18\,\mathbf{j}$	$\mathbf{r}'(3)$ is tangent to the curve
C)	$\mathbf{r}(3) = 20\,\mathbf{i} + 29\,\mathbf{j}$	$\mathbf{r}'(3) = 5\,\mathbf{i} + 18\,\mathbf{j}$	$\mathbf{r}(3)$ is tangent to the curve
D)	$\mathbf{r}(3) = 20\,\mathbf{i} + 29\,\mathbf{j}$	$\mathbf{r}'(3) = 5\,\mathbf{i} + 18\,\mathbf{j}$	$\mathbf{r}'(3)$ is normal to the curve
E)	$\mathbf{r}(3) = 20\,\mathbf{i} + 29\,\mathbf{j}$	$\mathbf{r}'(3) = 5\,\mathbf{i} + 54\,\mathbf{j}$	$\mathbf{r}'(3)$ is tangent to the curve

18. Find the vectors $\mathbf{r}(1)$ and $\mathbf{r}'(1)$ for the following vector function:

$$\mathbf{r}(t) = \left(1+4t\right)\,\mathbf{i} + \left(6+5t^2\right)\,\mathbf{j} + 4\,\mathbf{k}$$

	$\mathbf{r}(1)$	$\mathbf{r}'(1)$
A)	$\mathbf{r}(1) = 5\,\mathbf{i} + 11\,\mathbf{j} + 4\mathbf{k}$	$\mathbf{r}'(1) = 3\,\mathbf{i} + 10\,\mathbf{j}$
B)	$\mathbf{r}(1) = 5\,\mathbf{i} + 11\,\mathbf{j} + 4\mathbf{k}$	$\mathbf{r}'(1) = 4\,\mathbf{i} + 10\,\mathbf{j}$
C)	$\mathbf{r}(1) = 5\,\mathbf{i} + 4\,\mathbf{j} + 11\mathbf{k}$	$\mathbf{r}'(1) = 4\,\mathbf{i} + 10\,\mathbf{j}$
D)	$\mathbf{r}(1) = 5\,\mathbf{i} + 11\mathbf{k} + 4\,\mathbf{j}$	$\mathbf{r}'(1) = 4\,\mathbf{i} + 10\,\mathbf{j}$
E)	$\mathbf{r}(1) = 5\,\mathbf{i} + 11\,\mathbf{j} + 4\mathbf{k}$	$\mathbf{r}'(1) = 4\,\mathbf{i} + 9\,\mathbf{j}$

19. Find $\mathbf{r}'(t)$ given the following vector function:

$\mathbf{r}(t) = 4t^2\ \mathbf{i} + 3t^3\ \mathbf{j} + 2t^4\,\mathbf{k}$

A) $\mathbf{r}'(t) = 4t\ \ \mathbf{i} + 3t^2\ \mathbf{j} + 2t^3\,\mathbf{k}$

B) $\mathbf{r}'(t) = 8t\ \ \mathbf{i} + 9t^2\ \mathbf{j} + 8t^3\,\mathbf{k}$

C) $\mathbf{r}'(t) = 8t\ \ \mathbf{i} + 3t^2\ \mathbf{k} + 2t^3\mathbf{j}$

D) $\mathbf{r}'(t) = 4t\ \ \mathbf{i} + 3t^3\ \mathbf{j} + 2t^2\,\mathbf{k}$

E) $\mathbf{r}'(t) = 8t^2\ \mathbf{i} + 9t^3\ \mathbf{j} + 8t^4\,\mathbf{k}$

20. Find $\mathbf{r}'(t)$ given the following vector function:

$\mathbf{r}(t) = -5t^2\ \mathbf{i} - 4t^3\ \mathbf{j} + 5t^5\,\mathbf{k}$

A) $\mathbf{r}'(t) = -5t\ \ \mathbf{i} - 4t^2\ \mathbf{j} + 5t^4\,\mathbf{k}$

B) $\mathbf{r}'(t) = -10t\ \ \mathbf{i} - 12t^2\ \mathbf{j} + 25t^4\,\mathbf{k}$

C) $\mathbf{r}'(t) = -10t\ \ \mathbf{i} + 4t^2\ \mathbf{j} + 5t^4\,\mathbf{k}$

D) $\mathbf{r}'(t) = -5t\ \ \mathbf{i} - 4t^4\ \mathbf{j} + 5t^2\,\mathbf{k}$

E) $\mathbf{r}'(t) = -10t^2\ \mathbf{i} - 12t^3\ \mathbf{j} + 25t^5\,\mathbf{k}$

21. Find $\mathbf{r}'(t)$ given the following vector function:

$\mathbf{r}(t) = \dfrac{2}{t^2}\ \mathbf{i} - 5t^5\ \mathbf{j} + \dfrac{t^3}{4}\mathbf{k}$

A) $\mathbf{r}'(t) = -\dfrac{4}{t^3}\ \mathbf{i} - 25t^4\ \mathbf{j} + \dfrac{3}{4}t^2\,\mathbf{k}$

B) $\mathbf{r}'(t) = \dfrac{4}{t^3}\ \mathbf{i} - 25t^4\ \mathbf{j} + \dfrac{3}{4}t^2\,\mathbf{k}$

C) $\mathbf{r}'(t) = -\dfrac{4}{t^3}\ \mathbf{i} - 25t^5\ \mathbf{j} + \dfrac{3}{4}t^2\,\mathbf{k}$

D) $\mathbf{r}'(t) = \dfrac{4}{t^3}\ \mathbf{i} - 25t^4\ \mathbf{j} - \dfrac{3}{4}t^3\,\mathbf{k}$

E) $\mathbf{r}'(t) = -\dfrac{4}{t^3}\ \mathbf{i} - 25t^4\ \mathbf{j} + 12t^2\,\mathbf{k}$

22. Find $\mathbf{r'(}t)$ given the following vector function:

$\mathbf{r}(t) = 3\cos^3 t\ \mathbf{i} + 5\sin t^4\ \mathbf{j} + t^5\,\mathbf{k}$

A) $\mathbf{r}'(t) = -9\sin^2 t\cos t\ \mathbf{i} + 20t^3\cos t^4\sin t\,\mathbf{j} + 5t^4\,\mathbf{k}$

B) $\mathbf{r}'(t) = -9\sin^2 t\cos t\ \mathbf{i} - 20t^3\cos t^4\ \mathbf{j} + 5t^4\,\mathbf{k}$

C) $\mathbf{r}'(t) = 9\sin^2 t\cos t\ \mathbf{i} + 20t^3\cos t^4\ \mathbf{j} + 5t^4\,\mathbf{k}$

D) $\mathbf{r}'(t) = -9\sin^2 t\cos t\ \mathbf{i} + 20t^3\cos t^4\ \mathbf{j} + 5t^4\,\mathbf{k}$

E) $\mathbf{r}'(t) = -9\sin^2 t\ \mathbf{i} + 20t^3\cos t^4\ \mathbf{j} + 5t^4\,\mathbf{k}$

23. Find $\mathbf{r}'(t)\cdot\mathbf{r}''(t)$ given the following vector function:

$\mathbf{r}(t) = 2t^5\ \mathbf{i} - 5t^4\ \mathbf{j}$

A) 0

B) $400t^7 + 1200t^5$

C) $400t^7 - 1200t^5$

D) $400t^7\,\mathbf{i} + 1200t^5\,\mathbf{j}$

E) $400t^6 + 1200t^6$

24. Find $\mathbf{r}'(t)\cdot\mathbf{r}''(t)$ given the following vector function:

$\mathbf{r}(t) = \left(4t^2 + 4t\right)\mathbf{i} + \left(3t^2 + 5t\right)\mathbf{j}$

A) $62 + 50t$

B) $31 + 100t$

C) $62 + 100t$

D) $62 + 100t^2$

E) $31 + 50t$

25. Find $\mathbf{r}'(t)\cdot\mathbf{r}''(t)$ given the following vector function:

$\mathbf{r}(t) = 4\cos t\ \mathbf{i} + 6\sin t\,\mathbf{j}$

A) $-52\cos t \sin t$

B) $4\cos t \sin t$

C) $52\cos t \sin t$

D) $-20\cos t \sin t$

E) $\cos t \sin t$

26. Use the properties of the derivative to find $D_t[\mathbf{r}(t)\cdot\mathbf{u}(t)]$ given the following vector valued functions:

$\mathbf{r}(t) = 2t\ \mathbf{i} + 2t^3\mathbf{j} + 2t^3\mathbf{k}$

$\mathbf{u}(t) = 5\ \mathbf{i} + 5t^2\mathbf{j} - 3t^3\mathbf{k}$

A) $10 + 50t^4 - 36t^5$
B) $10 + 50t^4 + 36t^5$
C) $10 + 36t^4 - 21t^5$
D) $10t + 50t^4 - 36t^5$
E) $-10 + 50t^4 + 36t^5$

27. Use the properties of the derivative to find $D_t[[\$a]\mathbf{r}(t) - 5\mathbf{u}(t)]$ given the following vector valued functions:

$\mathbf{r}(t) = 2t\ \mathbf{i} + 5t^3\mathbf{j} + 3t^2\mathbf{k}$

$\mathbf{u}(t) = 2\ \mathbf{i} + 6t^2\mathbf{j} + 5t^3\mathbf{k}$

A) $6\ \mathbf{i} + \left(45t^3 - 60t\right)\mathbf{j} + \left(18t - 75t^2\right)\mathbf{k}$
B) $6\mathbf{i} + \left(45t^2 - 60t\right)\mathbf{j} + \left(18t - 75t^2\right)\mathbf{k}$
C) $6\ \mathbf{i} + \left(45t^2 + 60t\right)\mathbf{j} + \left(18t + 75t^2\right)\mathbf{k}$
D) $6t\ \mathbf{i} + \left(45t^2 + 60t\right)\mathbf{j} - \left(18t + 75t^2\right)\mathbf{k}$
E) $6\ \mathbf{i} + \left(45t^2 - 60t\right)\mathbf{j} + \left(18t^2 - 75t\right)\mathbf{k}$

28. Use the properties of the derivative to find $D_t[\mathbf{r}(t)\times\mathbf{u}(t)]$ given the following vector valued functions:

$$\mathbf{r}(t) = 4t\ \mathbf{i} - 4t^3\mathbf{j} + 4t^2\mathbf{k}$$

$$\mathbf{u}(t) = 2\ \mathbf{i} + 5t^2\mathbf{j} + 3t^3\mathbf{k}$$

A) $\left(-80t^3 + 72t^5\right)\mathbf{i} + \left(16t - 48t^3\right)\mathbf{j} - 36t^2\mathbf{k}$
B) $\left(-80t^3 - 72t^5\right)\mathbf{i} + \left(16t + 48t^3\right)\mathbf{j} + 84t^3\mathbf{k}$
C) $\left(-80t^3 - 72t^5\right)\mathbf{i} + \left(16t^2 - 48t^3\right)\mathbf{j} + 84t^2\mathbf{k}$
D) $\left(-80t^3 - 72t^5\right)\mathbf{i} + \left(48t + 16t^3\right)\mathbf{j} + 84t^2\mathbf{k}$
E) $\left(-80t^3 - 72t^5\right)\mathbf{i} + \left(16t - 48t^3\right)\mathbf{j} + 84t^2\mathbf{k}$

29. Use the properties of the derivative to find $D_t[\mathbf{r}(t)\cdot\mathbf{u}(t)]$ given the following vector valued functions:

$$\mathbf{r}(t) = t\ \mathbf{i} + 2\cos 2t\mathbf{j} + 2\sin 2t\mathbf{k}$$

$$\mathbf{u}(t) = \frac{4}{t}\ \mathbf{i} + 2\cos 2t\mathbf{j} + 2\sin 2t\mathbf{k}$$

A) $\frac{1}{t}$
B) 4
C) 0
D) 1
E) $1t$

30. Use the properties of the derivative to find $D_t[\mathbf{r}(t) \times \mathbf{u}(t)]$ given the following vector valued functions:

$$\mathbf{r}(t) = 2\cos 2t\ \mathbf{j} + 2\sin 2t\ \mathbf{k}$$

$$\mathbf{u}(t) = \frac{2}{t}\ \mathbf{i} + 2\cos 2t\ \mathbf{j} + 2\sin 2t\ \mathbf{k}$$

A) $\left(\frac{8\cos(2t)}{t} - \frac{4\sin(2t)}{t^2}\right)\mathbf{j} + \left(\frac{4\cos(2t)}{t^2} + \frac{8\sin(2t)}{t}\right)\mathbf{k}$

B) $\left(\frac{8\cos(2t)}{t} + \frac{4\sin(2t)}{t^2}\right)\mathbf{j} + \left(\frac{4\cos(2t)}{t^2} + \frac{8\sin(2t)}{t}\right)\mathbf{k}$

C) $\left(\frac{8\cos(2t)}{t} - \frac{4\sin(2t)}{t^2}\right)\mathbf{j} + \left(\frac{4\cos(2t)}{t^2} - \frac{8\sin(2t)}{t}\right)\mathbf{k}$

D) $\left(\frac{8\cos(2t)}{t} - \frac{4\sin(2t)}{t^2}\right)\mathbf{i} + \left(\frac{4\cos(2t)}{t^2} + \frac{8\sin(2t)}{t}\right)\mathbf{k}$

E) $\left(\frac{8\cos(2t)}{t^2} - \frac{4\sin(2t)}{t}\right)\mathbf{j} + \left(\frac{4\cos(2t)}{t^2} + \frac{8\sin(2t)}{t}\right)\mathbf{k}$

31. Use the properties of the derivative to find $D_t[\|\mathbf{r}(t)\|]$ given the vector valued function below.

$$\mathbf{r}(t) = 2t\ \mathbf{i} + 3\cos 4t\ \mathbf{j} + 3\sin 4t\ \mathbf{k}$$

A) $\frac{4}{\sqrt{9+4t^2}}$

B) $\frac{9t}{\sqrt{9+4t^2}}$

C) $\frac{4t}{\sqrt{9+9t^2}}$

D) $\frac{4t}{\sqrt{9+4t^2}}$

E) $\frac{9t}{\sqrt{9+4t^2}}$

32. Find the indefinite integral below.

$$\int \left(8t \ \mathbf{i} + 12t^5 \ \mathbf{j} + 15t^2 \ \mathbf{k} \right) dt$$

Do not include an arbitrary constant vector.

A) $4t^2 \ \mathbf{i} + 2t^5 \ \mathbf{j} + 5t^3 \ \mathbf{k}$
B) $4t^2 \ \mathbf{i} + 2t^6 \ \mathbf{j} + 15t^3 \ \mathbf{k}$
C) $4t^2 \ \mathbf{i} + 2t^6 \ \mathbf{j} + 5t^3 \ \mathbf{k}$
D) $4t^2 \mathbf{j} + 2t^6 \ \mathbf{i} + 5t^3 \ \mathbf{k}$
E) $4t^2 \ \mathbf{i} + 2t^2 \ \mathbf{j} + 5t^3 \ \mathbf{k}$

33. Find the indefinite integral below.

$$\int \left(\frac{-8}{t^3} \ \mathbf{i} + 12t^5 \ \mathbf{j} + 2t^{-1/3} \ \mathbf{k} \right) dt$$

Do not include an arbitrary constant vector.

A) $\frac{4}{t^2} \ \mathbf{i} + 2t^6 \ \mathbf{j} - 3t^{2/3} \ \mathbf{k}$
B) $-\frac{4}{t^2} \ \mathbf{i} + 2t^6 \ \mathbf{j} + 3t^{2/3} \ \mathbf{k}$
C) $\frac{4}{t^2} \ \mathbf{i} + 2t^6 \ \mathbf{j} + 3t^{2/3} \ \mathbf{k}$
D) $\frac{4}{t^3} \ \mathbf{i} + 2t^6 \ \mathbf{j} + 3t^{2/3} \ \mathbf{k}$
E) $\frac{4}{t^2} \ \mathbf{i} + 2t^6 \ \mathbf{j} + 3t^{3/2} \ \mathbf{k}$

34. Find the indefinite integral below.

$$\int \left(4e^{4t}\ \mathbf{i} - 2\sin 2t\ \mathbf{j} + 25\cos 5t\ \mathbf{k}\right)dt$$

Do not include an arbitrary constant vector.

A) $e^{4t}\,\mathbf{i} + \cos 2t\ \mathbf{j} + 25\sin 5t\ \mathbf{k}$

B) $\dfrac{e^{4t}}{4}\mathbf{i} + \cos 2t\ \mathbf{j} + 5\sin 5t\ \mathbf{k}$

C) $e^{4t}\,\mathbf{i} + \cos 2t\ \mathbf{j} - 5\sin 5t\ \mathbf{k}$

D) $e^{4t}\,\mathbf{i} + \cos 2t\ \mathbf{j} + 5\sin 5t\ \mathbf{k}$

E) $e^{4t}\,\mathbf{i} - \cos 2t\ \mathbf{j} + 5\sin 5t\ \mathbf{k}$

35. Evaluate the definite integral below.

$$\int_1^3 \left(-4t\ \ \mathbf{i} + 16t^3\ \mathbf{j} + 12t^2\ \mathbf{k}\right)dt$$

A) $-16\ \mathbf{i} + 320\ \mathbf{j} + 104\ \mathbf{k}$

B) $324\ \mathbf{i} + 320\ \mathbf{j} + 104\ \mathbf{k}$

C) $-16\ \mathbf{i} + 324\ \mathbf{j} + 104\ \mathbf{k}$

D) $-16\ \mathbf{i} + 320\ \mathbf{j} + 108\ \mathbf{k}$

E) $324\ \mathbf{i} + 324\ \mathbf{j} + 108\ \mathbf{k}$

36. Evaluate the definite integral below.

$$\int_1^2 \left(\frac{-15}{t^4}\ \mathbf{i} - 5\sin t\ \mathbf{j} + 3t^{-1/4}\ \mathbf{k}\right)dt$$

A) $\dfrac{35}{8}\ \mathbf{i} - 5(\cos 2 - \cos 1)\ \mathbf{j} + 3\left(2^{3/4} - 1\right)\mathbf{k}$

B) $\dfrac{35}{8}\ \mathbf{i} + 5(\cos 2 - \cos 1)\ \mathbf{j} - 3\left(2^{3/4} - 1\right)\mathbf{k}$

C) $\dfrac{5}{8}\ \mathbf{i} + 5(\cos 2 - \cos 1)\ \mathbf{j} + 3\left(2^{4/3} - 1\right)\mathbf{k}$

D) $-\dfrac{35}{8}\ \mathbf{i} + 5(\cos 2 - \cos 1)\ \mathbf{j} + 4\left(2^{3/4} - 1\right)\mathbf{k}$

E) $\dfrac{35}{8}\ \mathbf{i} + 5(\cos 2 - \cos 1)\ \mathbf{j} + 3\left(2^{3/4} - 1\right)\mathbf{k}$

37. Find **r**(t) given the following:

$\mathbf{r}'(t) = 15t^4\ \mathbf{j} + 8t\ \ \mathbf{k},\ \mathbf{r}(0) = -5\ \mathbf{i}\ + 15\ \mathbf{j}$

A) $\mathbf{r}(t) = -5\mathbf{i} + (15 - 3t^5)\,\mathbf{j} - 4t^2\ \mathbf{k}$

B) $\mathbf{r}(t) = (15 + 3t^5)\,\mathbf{j} + 4t^2\ \mathbf{k}$

C) $\mathbf{r}(t) = 15\mathbf{i} + (-5 + 3t^5)\,\mathbf{j} + 4t^2\ \mathbf{k}$

D) $\mathbf{r}(t) = -5\mathbf{i} + (15 + 3t^5)\,\mathbf{j} + 4t^2\ \mathbf{k}$

E) $\mathbf{r}(t) = -5\mathbf{i} + 4t^2\ \mathbf{j} + (15 + 3t^5)\mathbf{k}$

38. Find **r**(t) given the following information:

$\mathbf{r}''(t) = 4\ \mathbf{i}\ + 18t\ \mathbf{k},\quad \mathbf{r}'(0) = 4\,\mathbf{j}\ ,\quad \mathbf{r}(0) = -5\ \mathbf{i}$

A) $\mathbf{r}(t) = \left(2t^2 - 5\right)\mathbf{i} + 4t\ \mathbf{k}\ + 9t^3\ \mathbf{j}$

B) $\mathbf{r}(t) = \left(2t^2 - 5\right)\mathbf{i} + 4t\ \ \mathbf{j} + 9t^3\,\mathbf{k}$

C) $\mathbf{r}(t) = \left(2t^2 - 5\right)\mathbf{i} + 4t\ \ \mathbf{j} + 3t^3\,\mathbf{k}$

D) $\mathbf{r}(t) = \left(2t^2 - 5\right)\mathbf{i} + 4t\ \ \mathbf{j} + 9t^2\,\mathbf{k}$

E) $\mathbf{r}(t) = \left(2t^2 - 5\right)\mathbf{i} + 4t^2\ \ \mathbf{j} + 9t^3\,\mathbf{k}$

39. The position vector **r** describes the path of an object moving in the *xy*-plane. Find the velocity and acceleration vectors at the given point.

$\mathbf{r}(t) = 2t^4\mathbf{i} - 2t^3\mathbf{j},\quad (2,-2)$

A) $\mathbf{v}(t) = 8\ \mathbf{i}\ - 6\ \mathbf{j},\ \mathbf{a}(t) = 24\ \mathbf{i}\ - 12\ \mathbf{j}$

B) $\mathbf{v}(t) = 8\ \mathbf{i}\ - 3\ \mathbf{j},\quad \mathbf{a}(t) = 24\ \mathbf{i}\ - 12\ \mathbf{j}$

C) $\mathbf{v}(t) = 8\ \mathbf{i}\ - 6\ \mathbf{j},\ \mathbf{a}(t) = 3\ \mathbf{i}\ - 12\,\mathbf{j}$

D) $\mathbf{v}(t) = 0\ \mathbf{i}\ - 6\ \mathbf{j},\ \mathbf{a}(t) = 24\ \mathbf{i}\ - 3\ \mathbf{j}$

E) $\mathbf{v}(t) = 0\ \mathbf{i}\ - 3\ \mathbf{j},\ \mathbf{a}(t) = 3\ \mathbf{i}\ - 3\ \mathbf{j}$

40. The position vector **r** describes the path of an object moving in the *xy*-plane. Find the velocity and acceleration vectors at the given point.

$\mathbf{r}(t) = \langle -2t - 3\cos t, -2 - 5\sin t \rangle, \quad t = 3$

A) $\mathbf{v}(t) = \langle -1.58, 4.95 \rangle,\ \mathbf{a}(t) = \langle 1.25, 0.71 \rangle,$

B) $\mathbf{v}(t) = \langle -1.58, 2.08 \rangle,\ \mathbf{a}(t) = \langle -2.97, 0.71 \rangle,$

C) $\mathbf{v}(t) = \langle -1.58, 4.95 \rangle,\ \mathbf{a}(t) = \langle -2.97, -0.71 \rangle,$

D) $\mathbf{v}(t) = \langle -1.58, 4.95 \rangle,\ \mathbf{a}(t) = \langle -2.97, 0.71 \rangle,$

E) $\mathbf{v}(t) = \langle 0.73, 4.95 \rangle,\ \mathbf{a}(t) = \langle -2.97, 0.71 \rangle,$

41. The position vector **r** describes the path of an object moving in space. Find the velocity, speed, and acceleration of the object.

$\mathbf{r}(t) = 2t\mathbf{i} + (5t - 3)\mathbf{j} + (4 - 3t)\mathbf{k}$

	Velocity	Speed	Acceleration
A)	$2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$	$\sqrt{38}$	$\mathbf{0}$
B)	$2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$	$\sqrt{38}$	0
C)	$2\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$	$\sqrt{38}$	0
D)	$2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$	$\sqrt{38}$	$\mathbf{i} + \mathbf{j} - \mathbf{k}$
E)	$5\mathbf{j} - 3\mathbf{k}$	$\sqrt{34}$	0

42. The position vector **r** describes the path of an object moving in space. Find the velocity, speed, and acceleration of the object.

$$\mathbf{r}(t) = -2\,\mathbf{i} + 4t^2\,\mathbf{j} - 6t^2\,\mathbf{k}$$

	Velocity	Speed	Acceleration
A)	$4\,\mathbf{j} - 6\,\mathbf{k}$	$\sqrt{52}t$	$\mathbf{0}$
B)	$8\,t\,\mathbf{j} - 12t\,\mathbf{k}$	$\sqrt{208}t$	$8\,\mathbf{j} - 12\,\mathbf{k}$
C)	$8\,t\,\mathbf{j} - 12t\,\mathbf{k}$	$\sqrt{208t}$	$8\,\mathbf{j} - 12\,\mathbf{k}$
D)	$8\,t\,\mathbf{j} + 12t\,\mathbf{k}$	$\sqrt{208}t$	$\mathbf{i} + \mathbf{j} - \mathbf{k}$
E)	$8\,t\,\mathbf{j} + 12t\,\mathbf{k}$	$\sqrt{208}t$	$8\,\mathbf{j} + 12\,\mathbf{k}$

43. The position vector **r** describes the path of an object moving in space. Find the velocity, speed, and acceleration of the object.

$\mathbf{r}(t) = \left\langle e^{4t} \sin t, e^{4t} \cos t, 3e^{4t} \right\rangle$

A) velocity $= \left\langle e^{4t}(4\sin t - \cos t), e^{4t}(4\cos t + \sin t), 12e^{4t} \right\rangle$
speed $= e^{4t}\sqrt{161}$
acceleration $= \left\langle e^{4t}(8\cos t + 15\sin t), e^{4t}(-8\sin t + 15\cos t), 48e^{4t} \right\rangle$

B) velocity $= \left\langle e^{4t}(4\sin t + \cos t), e^{4t}(4\cos t + \sin t), 12e^{4t} \right\rangle$
speed $= e^{4t}\sqrt{161}$
acceleration $= \left\langle e^{4t}(8\cos t + 15\sin t), e^{4t}(-8\sin t + 15\cos t), 48e^{4t} \right\rangle$

C) velocity $= \left\langle e^{4t}(4\sin t + \cos t), e^{4t}(4\cos t - \sin t), 12e^{4t} \right\rangle$
speed $= e^{4t}\sqrt{161}$
acceleration $= \left\langle e^{4t}(8\cos t + 15\sin t), e^{4t}(-8\sin t + 15\cos t), 48e^{4t} \right\rangle$

D) velocity $= \left\langle e^{4t}(4\sin t + \cos t), e^{4t}(4\cos t + \sin t), 12e^{4t} \right\rangle$
speed $= e^{4t}\sqrt{161}$
acceleration $= \left\langle e^{4t}(8\cos t + 15\sin t), -e^{4t}(-8\sin t + 15\cos t), 48e^{4t} \right\rangle$

E) velocity $= \left\langle e^{4t}(4\sin t + \cos t), -e^{4t}(4\cos t + \sin t), 12e^{4t} \right\rangle$
speed $= e^{4t}\sqrt{161}$
acceleration $= \left\langle e^{4t}(8\cos t + 15\sin t), e^{4t}(-8\sin t + 15\cos t), 48e^{4t} \right\rangle$

44. Use the given acceleration function to find the velocity and position vector. Then find the position at time t = 3.

$\mathbf{a}(t) = 4\mathbf{i} + 10\mathbf{j} + 8\mathbf{k}, \quad \mathbf{v}(0) = \mathbf{0}, \quad \mathbf{r}(0) = 5\mathbf{j}$

A) $\mathbf{r}(3) = 18\mathbf{i} + 50\mathbf{j} + 36\mathbf{k}$
B) $\mathbf{r}(3) = 18\mathbf{i} + 50\mathbf{j} - 36\mathbf{k}$
C) $\mathbf{r}(3) = 24\mathbf{i} + 50\mathbf{j} + 36\mathbf{k}$
D) $\mathbf{r}(3) = 18\mathbf{i} + 46\mathbf{j} - 36\mathbf{k}$
E) $\mathbf{r}(3) = 18\mathbf{i} + 50\mathbf{j} + 41\mathbf{k}$

45. Use the given acceleration function to find the velocity and position vector. Then find the position at time t = 3.

$\mathbf{a}(t) = -6\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}, \quad \mathbf{v}(0) = 4\mathbf{k}, \quad \mathbf{r}(0) = \mathbf{0}$

A) $\mathbf{r}(3) = -27\mathbf{i} + 22\mathbf{j} - 39\mathbf{k}$
B) $\mathbf{r}(3) = -27\mathbf{i} + 18\mathbf{j} - 39\mathbf{k}$
C) $\mathbf{r}(3) = -15\mathbf{i} + 18\mathbf{j} + 39\mathbf{k}$
D) $\mathbf{r}(3) = -27\mathbf{i} + 18\mathbf{j} + 39\mathbf{k}$
E) $\mathbf{r}(3) = -27\mathbf{i} + 18\mathbf{j} + 27\mathbf{k}$

46. Use the given acceleration function to find the velocity and position vector. Then find the position at time t = 1.

$\mathbf{a}(t) = 4\cos t\,\mathbf{i} - 2\sin t\,\mathbf{j}, \quad \mathbf{v}(0) = 6\mathbf{j} + 2\mathbf{k}, \quad \mathbf{r}(0) = -4\mathbf{i}$

A) $\mathbf{r}(1) = -4\cos 1\,\mathbf{i} + (2\sin 1 - 4)\mathbf{j} + 2\mathbf{k}$
B) $\mathbf{r}(1) = -4\cos 1\,\mathbf{i} + 2\mathbf{j} + (2\sin 1 + 4)\mathbf{k}$
C) $\mathbf{r}(1) = 4\cos 1\,\mathbf{i} + (2\sin 1 + 4)\mathbf{j} + 2\mathbf{k}$
D) $\mathbf{r}(1) = -4\cos 1\,\mathbf{i} - (2\sin 1 + 4)\mathbf{j} + 2\mathbf{k}$
E) $\mathbf{r}(1) = -4\cos 1\,\mathbf{i} + (2\sin 1 + 4)\mathbf{j} + 2\mathbf{k}$

47. Find the vector valued function for the path of a projectile launched at a height of 20 feet above the ground with an initial velocity of 65 feet per second at an angle of 40 degrees above the horizontal. Use a graphing utility to graph the path of the projectile and confirm your selection.

Let **i** be the unit vector in the horizontal direction and **j** the unit vector in the vertical direction.

A) $\mathbf{r}(t) = 65\sin 40t\ \mathbf{i} + \left(20 + 65\cos 40t - \frac{1}{2}gt^2\right)\mathbf{j}$
B) $\mathbf{r}(t) = 65\cos 40t\ \mathbf{i} + \left(20 + 65\sin 40t - \frac{1}{2}gt^2\right)\mathbf{j}$
C) $\mathbf{r}(t) = 65\cos 40t\ \mathbf{i} + \left(20 - 65\sin 40t - \frac{1}{2}gt^2\right)\mathbf{j}$
D) $\mathbf{r}(t) = 65\cos 40t\,\mathbf{j} + \left(20 + 65\sin 40t - \frac{1}{2}gt^2\right)\mathbf{i}$
E) $\mathbf{r}(t) = 65\cos 40t\ \mathbf{i} - \left(20 + 65\sin 40t - \frac{1}{2}gt^2\right)\mathbf{j}$

48. Determine the maximum height and range of a projectile fired at a height of 12 feet above the ground with an initial velocity of 100 feet per second at an angle of 25 degrees above the horizontal.

	Maximum height	Range
A)	153.61 feet	262.83 feet
B)	14.74 feet	81.99 feet
C)	39.91 feet	262.83 feet
D)	39.91 feet	256.69 feet
E)	39.91 feet	256.69 feet

49. Find the unit tangent vector to the curve given below at the specified point.

$\mathbf{r}(t) = -3t\ \mathbf{i} - 5t^2\mathbf{j}, \quad t = 3$

A) $\mathbf{T}(3) = \frac{-3}{\sqrt{909}}\ \mathbf{i} - \frac{30}{\sqrt{909}}\ \mathbf{j}$

B) $\mathbf{T}(3) = \frac{-5}{\sqrt{909}}\ \mathbf{i} - \frac{30}{\sqrt{909}}\ \mathbf{j}$

C) $\mathbf{T}(3) = \frac{-3}{\sqrt{909}}\ \mathbf{i} - \frac{5}{\sqrt{909}}\ \mathbf{j}$

D) $\mathbf{T}(3) = \frac{-3}{\sqrt{909}}\ \mathbf{j} - \frac{30}{\sqrt{909}}\mathbf{i}$

E) $\mathbf{T}(3) = \frac{-3}{\sqrt{909}}\ \mathbf{i} - \frac{30}{\sqrt{234}}\ \mathbf{j}$

50. Find the unit tangent vector to the curve given below at the specified point.

$\mathbf{r}(t) = -4\cos t\ \mathbf{i} + 4\sin t\ \mathbf{j}, \quad t = \frac{5\pi}{3}$

A) $\mathbf{T}\left(\frac{5\pi}{3}\right) = -0.50\mathbf{i} + 0.50\ \mathbf{j}$

B) $\mathbf{T}\left(\frac{5\pi}{3}\right) = -0.87\mathbf{i} + 0.87\ \mathbf{j}$

C) $\mathbf{T}\left(\frac{5\pi}{3}\right) = -0.87\ \mathbf{j} + 0.50\mathbf{i}$

D) $\mathbf{T}\left(\frac{5\pi}{3}\right) = -0.50\mathbf{i} + 0.87\ \mathbf{j}$

E) $\mathbf{T}\left(\frac{5\pi}{3}\right) = -0.87\mathbf{i} + 0.50\ \mathbf{j}$

51. Find the unit tangent vector **T(***t***)** and then use it to find a set of parametric equations for the line tangent to the space curve given below at the given point.

$\mathbf{r}(t) = -4t\,\mathbf{i} + 5t^2\,\mathbf{j} + 4t\mathbf{k}, \quad t = 4$

A) $x = -16 - 4s,\ \ y = 80 - 40s,\ \ z = 16 + 4s$

B) $x = -16 - 4s,\ \ y = 80 + 40s,\ \ z = 16 + 4s$

C) $x = -16 - 4s,\ \ y = 80 - 40s,\ \ z = 16 - 4s$

D) $x = -16 - 4s,\ \ y = 80 + 40s,\ \ z = 16 - 4s$

E) $x = 16 + 4s,\ \ y = 80 + 40s,\ \ z = -16 - 4s$

52. Find a set of parametric equations for the tangent line to the graph at $t = 2$ and use the equations for the line to approximate **r**(2+0.1).

$\mathbf{r}(t) = \left\langle e^{-3t}, -2\cos t, 2\sin t \right\rangle$

A) $\mathbf{r}(2.1) = \langle 0.00, 1.01, 1.90 \rangle$

B) $\mathbf{r}(2.1) = \langle 0.00, 1.01, 1.74 \rangle$

C) $\mathbf{r}(2.1) = \langle 0.00, 0.65, 1.90 \rangle$

D) $\mathbf{r}(2.1) = \langle 0.00, 0.65, 1.74 \rangle$

E) $\mathbf{r}(2.1) = \langle 0.00, 1.01, 1.74 \rangle$

53. Verify that the space curves intersect at the given values of the parameter. Find the angle between the tangent vectors at the point of intersection.

$$\mathbf{r}(t) = \langle t^3 - 3t, -2t^2, t \rangle, \qquad t = 0$$
$$\mathbf{u}(t) = \langle 5s,\ s, \sin s \rangle, \qquad s = 0$$

A) 133.43°
B) 145.43°
C) 148.43°
D) 151.43°
E) 169.43°

54. Find the principle unit normal vector to the curve given below at the specified point.

$$\mathbf{r}(t) = t\ \mathbf{i} + 5t^2\ \mathbf{j}, \quad t = 1$$

A) $\mathbf{N}(1) = \dfrac{-10}{\sqrt{101}}\ \mathbf{i} + \dfrac{1}{\sqrt{101}}\ \mathbf{j}$
B) $\mathbf{N}(1) = \dfrac{-10}{\sqrt{26}}\ \mathbf{i} - \dfrac{1}{\sqrt{26}}\ \mathbf{j}$
C) $\mathbf{N}(1) = \dfrac{-10}{\sqrt{101}}\ \mathbf{i} - \dfrac{1}{\sqrt{101}}\ \mathbf{j}$
D) $\mathbf{N}(1) = \dfrac{-10}{\sqrt{26}}\ \mathbf{i} + \dfrac{1}{\sqrt{26}}\ \mathbf{j}$
E) None of the above

55. Find the principle unit normal vector to the curve given below at the specified point.

$$\mathbf{r}(t) = t\ \mathbf{i} + \frac{2}{t}\mathbf{j}, \quad t = 3$$

A) $\mathbf{N}(3) = \dfrac{2}{\sqrt{325}}\ \mathbf{i} + \dfrac{9}{\sqrt{325}}\ \mathbf{j}$
B) $\mathbf{N}(3) = \dfrac{2}{\sqrt{325}}\ \mathbf{i} - \dfrac{9}{\sqrt{325}}\ \mathbf{j}$
C) $\mathbf{N}(3) = \dfrac{2}{\sqrt{85}}\ \mathbf{i} - \dfrac{9}{\sqrt{85}}\ \mathbf{j}$
D) $\mathbf{N}(3) = \dfrac{2}{\sqrt{85}}\ \mathbf{i} + \dfrac{9}{\sqrt{85}}\ \mathbf{j}$
E) None of the above

56. Find the principle unit normal vector to the curve given below at the specified point.

$$\mathbf{r}(t) = 6\cos t\ \mathbf{i} + 6\sin t\ \mathbf{j}, \quad t = \frac{1}{2}\pi$$

A) $\mathbf{N} = \langle 1.00, 0.00 \rangle$
B) $\mathbf{N} = \langle 0.00, 0.00 \rangle$
C) $\mathbf{N} = \langle -1.00, 1.00 \rangle$
D) $\mathbf{N} = \langle 1.00, -1.00 \rangle$
E) $\mathbf{N} = \langle 0.00, -1.00 \rangle$

57. Find $\mathbf{v}(t)$, $\mathbf{a}(t)$, $\mathbf{T}(t)$ and $\mathbf{N}(t)$ (if it exists) for an object moving along the path given by the vector valued function given below. Use the results to determine the path. Is the speed of the object constant or changing?

$$\mathbf{r}(t) = -3t^2\mathbf{j} + 4\mathbf{k}$$

A) **N** is undefined. The path is a line and the speed is constant.
B) **N** is undefined. The path is a line and the speed is variable.
C) **N = i** . The path is a line and the speed is variable.
D) **T**= 0. The path is a line and the speed is constant.
E) **N** is undefined. The path is a line and the acceleration is variable.

58. Find $\mathbf{T}(t)$, $\mathbf{N}(t)$, $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$ at the given time t for the plane curve given below.

$$\mathbf{r}(t) = \left(t^3 - 2t\right)\mathbf{i} + \left(5t^2 - 4\right)\mathbf{j}, \quad t = 0$$

A) $\mathbf{T} = \langle -1, 0 \rangle$, $\mathbf{N} = \langle 0, 1 \rangle$, $a_{\mathbf{T}} = \mathbf{0}$, $a_{\mathbf{N}} = 10$
B) $\mathbf{T} = \langle 1, 0 \rangle$, $\mathbf{N} = \langle 0, 1 \rangle$, $a_{\mathbf{T}} = 10$, $a_{\mathbf{N}} = 0$
C) $\mathbf{T} = \langle -1, 0 \rangle$, $\mathbf{N} = \langle 0, 1 \rangle$, $a_{\mathbf{T}} = 0$, $a_{\mathbf{N}} = -10$
D) $\mathbf{T} = \langle 1, 0 \rangle$, $\mathbf{N} = \langle 0, 1 \rangle$, $a_{\mathbf{T}} = 0$, $a_{\mathbf{N}} = 10$
E) $\mathbf{T} = \langle -1, 0 \rangle$, $\mathbf{N} = \langle 0, 1 \rangle$, $a_{\mathbf{T}} = 0$, $a_{\mathbf{N}} = 10$

59. Find $\mathbf{T}(t)$, $\mathbf{N}(t)$, and hence determine $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$ at $t=0$ for the plane curve given below.

$\mathbf{r}(t) = e^{4t}\,\mathbf{i} + e^{-4t}\,\mathbf{j} + 3t\mathbf{k}$

A) $a_{\mathbf{T}} = \mathbf{0},\ a_{\mathbf{N}} = 16\sqrt{2}$
B) $a_{\mathbf{T}} = 0,\ a_{\mathbf{N}} = 16\sqrt{2}$
C) $a_{\mathbf{T}} = 16\sqrt{2},\ a_{\mathbf{N}} = 0$
D) $a_{\mathbf{T}} = 0,\ a_{\mathbf{N}} = 16$
E) $a_{\mathbf{T}} = 16,\ a_{\mathbf{N}} = \mathbf{0}$

60. Find $\mathbf{T}(t)$, $\mathbf{N}(t)$, and hence determine $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$ at $t=0$ for the space curve given below.

$\mathbf{r}(t) = e^{6t}\sin t\,\mathbf{i} + e^{6t}\cos t\,\mathbf{j} + e^{6t}\,\mathbf{k}$

A) $a_{\mathbf{T}} = \sqrt{37},\ a_{\mathbf{N}} = 37\sqrt{73}$
B) $a_{\mathbf{T}} = \sqrt{37},\ a_{\mathbf{N}} = 6\sqrt{73}$
C) $a_{\mathbf{T}} = 6\sqrt{73},\ a_{\mathbf{N}} = 0$
D) $a_{\mathbf{T}} = 6\sqrt{73},\ a_{\mathbf{N}} = \sqrt{37}$
E) $a_{\mathbf{T}} = 0,\ a_{\mathbf{N}} = \sqrt{37}$

61. Find the length of the plane curve given below.

$\mathbf{r}(t) = 5\,\mathbf{i} + 4t^3\,\mathbf{j}, \quad [0,5]$

A) 50
B) 125
C) 166
D) 500
E) 116

62. Find the length of the plane curve given below.

$\mathbf{r}(t) = 5t\ \mathbf{i} + 5t^2\ \mathbf{j}, \quad [0,3]$

A) $\frac{15}{2}\sqrt{37} + \frac{5}{4}\operatorname{arcsinh} 3$

B) $\frac{15}{2}\sqrt{37} + \frac{5}{4}\operatorname{arcsinh} 6$

C) $\frac{15}{2}\sqrt{37} - \frac{5}{4}\operatorname{arcsinh} 6$

D) $\frac{15}{2}\sqrt{15} + \frac{5}{4}\operatorname{arcsinh} 6$

E) $\frac{15}{2}\sqrt{37} + \frac{5}{4}\operatorname{arccosh} 6$

63. Find the length of the plane curve given below.

$\mathbf{r}(t) = 4\cos t\ \mathbf{i} + 4\sin t\ \mathbf{j}, \quad [0,2]$

A) 9
B) 8
C) 7
D) 6
E) 11

64. Find the length of the space curve given below.

$\mathbf{r}(t) = 3\ \mathbf{i} + 2\cos t\ \mathbf{j} + 2\ \sin t\ \mathbf{k}, \qquad [0,3]$

A) $\sqrt{13}$
B) $3\sqrt{5}$
C) $3\sqrt{13}$
D) $\sqrt{5}$
E) 3

65. Find the curvature, ***K***, where *s* is the arc length parameter.

$$\mathbf{r}(s) = 7\cos\left(\frac{3s}{\sqrt{490}}\right)\mathbf{i} + 7\sin\left(\frac{3s}{\sqrt{490}}\right)\mathbf{j} + \frac{7}{\sqrt{490}}s\,\mathbf{k}$$

A) $\frac{7}{70}$
B) $\frac{3}{70}$
C) $\frac{9}{70}$
D) $\frac{29}{70}$
E) 0

66. Find the curvature, ***K***, of the plane curve at the given point.

$$\mathbf{r}(t) = (3t+6)\ \mathbf{i} + (2t^2-3)\ \mathbf{j}, \quad t = -1$$

A) $\frac{6}{13^{3/2}}$
B) $\frac{12}{13^{3/2}}$
C) $\frac{6}{25^{3/2}}$
D) $\frac{12}{25^{3/2}}$
E) $\frac{25}{12^{3/2}}$

67. Find the curvature, ***K***, of the plane curve at the given point.

$$\mathbf{r}(t) = -5t\ \mathbf{i} + \frac{2}{t}\mathbf{j}, \quad t = -1$$

A) $\dfrac{1}{29^{3/2}20}$

B) $\dfrac{10}{29^{3/2}}$

C) $\dfrac{20}{29^{3/2}}$

D) $\dfrac{20}{1^{3/2}}$

E) $\dfrac{20}{29^{1/2}}$

68. Find the curvature, ***K***, of the curve given below.

$$\mathbf{r}(t) = 2t\ \mathbf{i} + 3\cos t\ \mathbf{j}, \quad t = -2$$

A) $\dfrac{2\cos(-2)}{\left(4+9\sin^2(-2)\right)^{3/2}}$

B) $\dfrac{6\sin(-2)}{\left(4+9\cos^2(-2)\right)^{3/2}}$

C) $-\dfrac{6\cos(-2)}{\left(4+9\sin^2(-2)\right)^{3/2}}$

D) $\dfrac{6\cos(-2)}{\left(4+9\sin^2(-2)\right)^{3/2}}$

E) $\dfrac{6\cos^2(-2)}{\left(4+9\sin^2(-2)\right)^{3/2}}$

69. Find the curvature, ***K***, of the curve given below.

$\mathbf{r}(t) = t\ \mathbf{i} + 6t^2\mathbf{j} + 6t\,\mathbf{k}$

A) $12\sqrt{\dfrac{37}{\left(37+144t^2\right)^3}}$

B) $\dfrac{12\sqrt{37}}{\left(37+12t^2\right)^{3/2}}$

C) $\dfrac{6}{\sqrt{\left(37+144t^2\right)^3}}$

D) $\dfrac{12}{\sqrt{\left(37+144t^2\right)^3}}$

E) $\dfrac{12}{\sqrt{\left(6+144t^2\right)^3}}$

70. Find the curvature and the radius of curvature of the plane curve at the given point.

$y = \sqrt{25 - x^2}, \qquad x = 3$

A) $K = 25$ $\qquad R = 1/25$

B) $K = 1/25$ $\qquad R = 25$

C) $K = 1$ $\qquad R = 1$

D) $K = 1/5$ $\qquad R = 5$

E) $K = 5$ $\qquad R = 1/5$

71. (a) Find the point on the curve given below at which the curvature, ***K***, is maximum.

$y = 5x^2 + 3x$

(b) Find the limit of ***K*** as $x \to \infty$.

A) (a) $x = -\frac{3}{10}$ (b) $K \to \infty$ as $x \to \infty$

B) (a) $x = -\frac{3}{5}$ (b) $K \to 0$ as $x \to \infty$

C) (a) $x = \frac{3}{10}$ (b) $K \to 0$ as $x \to \infty$

D) (a) $x = -\frac{3}{10}$ (b) $K \to 0$ as $x \to \infty$

E) (a) $x = \frac{3}{10}$ (b) $K \to \infty$ as $x \to \infty$

72. Find the point on the curve given below at which the curvature, ***K***, is zero

$y = 8x^3 + 25x^2 + 2x$

A) $x = -\frac{25}{8}$

B) $x = \frac{24}{25}$

C) $x = -\frac{24}{25}$

D) $x = -\frac{25}{24}$

E) $x = \frac{25}{24}$

Answer Key

1. D
2. A
3. C
4. B
5. E
6. D
7. D
8. C
9. A
10. E
11. B
12. D
13. C
14. B
15. D
16. D
17. B
18. B
19. B
20. B
21. A
22. D
23. B
24. C
25. D
26. A
27. B
28. E
29. C
30. A
31. D
32. C
33. C
34. D
35. A
36. D
37. D
38. C
39. A
40. D
41. A
42. B
43. C
44. A

45. D
46. E
47. B
48. C
49. A
50. E
51. B
52. B
53. C
54. A
55. D
56. E
57. B
58. E
59. B
60. D
61. D
62. B
63. B
64. C
65. C
66. D
67. C
68. D
69. A
70. D
71. D
72. D