**Chapter 10 Formulas**

**Let P=($x_0$,$y_0$,$z_0$) and Q=($x_1$,$y_1$,$z_1$) be points in 3-Space; u = <$u_1$,$u_2$,$u_3$>, v = <$v_1$,$v_2$,$v_3$>, w = <$w_1$,$w_2$,$w_3$>, n = <a,b,c>, $n_1$, and $n_2$ be vectors in 3-Space. Vectors are shown in bold for the questions. Use the arrow notation for vectors in your answers.**

Give the formulas for the following:

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| The dot product of **u** and **v**: $\vec{u}\cdot\vec{v} = u_1v_1+u_2v_2+u_3v_3$ | The cosine of the angle between **u** and **v**: $\cos(\theta) = \dfrac{\vec{u}\cdot\vec{v}}{\lVert\vec{u}\rVert\,\lVert\vec{v}\rVert}$ |
| **u** and **v** are orthogonal: $\vec{u}\cdot\vec{v} = 0$ | The projection of **u** onto **v**: $\text{proj}_{\vec{v}}(\vec{u}) = \dfrac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\,\vec{v}$ |
| The norm of **v**: $\lVert\vec{v}\rVert = \sqrt{\vec{v}\cdot\vec{v}} = \sqrt{v_1^2+v_2^2+v_3^2}$ | The vector component of **u** orthogonal to **v**: $\vec{u} - \text{proj}_{\vec{v}}(\vec{u}) = \vec{u} - \dfrac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\,\vec{v}$ |
| The unit vector in the direction of **v**: $\dfrac{\vec{v}}{\lVert\vec{v}\rVert}$ | The cross product of **u** and **v**. $\vec{u}\times\vec{v} = (u_2v_3-u_3v_2)\vec{i} + (u_3v_1-u_1v_3)\vec{j} + (u_1v_2-u_2v_1)\vec{k}$ |

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| The vector equation of a line through P and parallel to <a,b,c>:<br>$\vec{r}(t) = \langle x,y,z\rangle$<br>$= \langle x_0,y_0,z_0\rangle + t\langle a,b,c\rangle$ | The standard equation of a plane through P with normal vector <a,b,c>:<br>$a(x-x_0)+b(y-y_0)$<br>$+c(z-z_0)=0$ |
| The parametric equations of a line through P and parallel to **v**:<br>$x=x_0+at$<br>$y=y_0+bt$<br>$z=z_0+ct$ | The cosine of the angle between two planes with normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$:<br>$\cos(\theta)=\dfrac{\lvert\vec{n}_1\cdot\vec{n}_2\rvert}{\lVert\vec{n}_1\rVert\,\lVert\vec{n}_2\rVert}$<br>**REVISED**<br>7:59 am, 3/12/06 |
| The vector equation of a line through P and Q:<br>$\langle x,y,z\rangle$<br>$=(1-t)\langle x_0,y_0,z_0\rangle$<br>$+t\langle x_1,y_1,z_1\rangle$ | The distance between the point Q and a plane through P with normal vector **n**:<br>$\dfrac{\lvert(\langle x_1,y_1,z_1\rangle-\langle x_0,y_0,z_0\rangle)\cdot\vec{n}\rvert}{\lVert\vec{n}\rVert}$ |
| The vector equation of a plane through P with normal vector **n** = <a,b,c>:<br>$(\langle x,y,z\rangle-\langle x_0,y_0,z_0\rangle)\cdot\vec{n}=0$ | The distance between the point Q and a line through P with direction vector **u**:<br>$\dfrac{\lVert(\langle x_1,y_1,z_1\rangle-\langle x_0,y_0,z_0\rangle)\times\vec{n}\rVert}{\lVert\vec{n}\rVert}$<br>**REVISED**<br>8:00 am, 3/12/06 |
| The volume of the parallelepiped with vectors **u**, **v**, and **w** as adjacent edges:<br>$\vec{u}\cdot(\vec{v}\times\vec{w})$<br>$=\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$ | The triple scalar product of **u**, **v**, and **w**:<br>$\vec{u}\cdot(\vec{v}\times\vec{w})$<br>$=\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$ |

**Chapter 11 Formulas**

Let f(t) be a real valued function of t; **r**(t) = <x(t),y(t),z(t)> and **u**(t) be vector valued functions; **r**(t) be the position vector, **v**(t) be the velocity, and **a**(t) be the acceleration; C be a smooth curve given by **r**(t) = x(t)**i** + y(t)**j** + z(t)**k** on the interval (a,b).

Give the formulas for the following:

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| **r**(t) is continuous at the point t = a: $\lim_{t \to a} \vec{r}(t) = \vec{r}(a)$ | $D_t$ [**r**(f(t))] = $f'(t)\,\vec{r}\,'(f(t))$ |
| The derivative of **r**(t): $\vec{r}\,'(t) = \lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}$ | If **r**(t) · **r**(t) = constant then $\vec{r} \cdot \vec{r}\,' = 0$ <br> REVISED 8:01 am, 3/12/06 |
| $D_t$ [**r**(t) + **u**(t)] = $\vec{r}\,'(t) + \vec{u}\,'(t)$ | $\int \vec{r}(t)dt =$ $\left(\int x(t)dt\right)\vec{i} + \left(\int y(t)dt\right)\vec{j} + \left(\int z(t)dt\right)\vec{k}$ |
| $D_t$ [**r**(t) · **u**(t)] = $\vec{r}\,' \cdot \vec{u} + \vec{r} \cdot \vec{u}\,'$ | Velocity: **v**(t) = $\vec{r}\,'(t)$ |
| $D_t$ [**r**(t) × **u**(t)] = $\vec{r}\,' \times \vec{u} + \vec{r} \times \vec{u}\,'$ | Acceleration: **a**(t) = $\vec{v}\,'(t) = \vec{r}\,''(t)$ |
| $D_t$ [f(t)**r**(t)] = $f'(t)\vec{r}(t) + f(t)\vec{r}\,'(t)$ | Speed = $\|\vec{v}(t)\| = \|\vec{r}\,'(t)\|$ |

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| Projectile position function for an initial velocity $\mathbf{v}_0$ and an initial position $\mathbf{r}_0$: $\mathbf{r}(t) =$ <br> $\vec{r}(t) = -\frac{1}{2}gt^2\vec{j} + t\vec{v}_0 + \vec{r}_0$ | The arc length of C: s = <br> $\int_a^b \|\vec{r}'(t)\| dt = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$ |
| The unit tangent vector: $\mathbf{T}(t) =$ <br> $\frac{\vec{r}'(t)}{\|\vec{r}'(t)\|}$ | The arc length function on C: $s(t) =$ <br> $\int_a^t \|\vec{r}'(u)\| du$ <br> REVISED 8:28 am, 3/12/06 |
| Principle unit normal vector: $\mathbf{N}(t) =$ <br> $\frac{\vec{T}'(t)}{\|\vec{T}'(t)\|}$ | The curvature for C given by the arc length parameterization $\mathbf{r}(s)$: K = <br> $\|\vec{T}'(s)\| = \|\vec{r}''(s)\|$ |
| Acceleration as a linear combination of **T** and **N**: <br> $\vec{a} = a_T\vec{T} + a_N\vec{N}$ | The curvature for C given by $\mathbf{r}(t)$: K = <br> $\frac{\|\vec{T}'(t)\|}{\|\vec{r}'(t)\|} = \frac{\|\vec{r}' \times \vec{r}''\|}{\|\vec{r}'\|^3}$ <br> REVISED 8:02 am, 3/12/06 |
| The tangential component of acceleration: $a_T =$ <br> $D_t\|\vec{v}(t)\| = \frac{\vec{r}' \cdot \vec{r}''}{\|\vec{r}'\|}$ | Acceleration in terms of speed (ds/dt) and curvature: <br> $\vec{a}(t) = \frac{d^2s}{dt^2}\vec{T} + K\left(\frac{ds}{dt}\right)^2\vec{N}$ |
| The normal or centripetal component of acceleration: $a_N =$ <br> REVISED 8:01 am, 3/12/06 <br> $\sqrt{\|\vec{r}''\|^2 - a_T^2} = \frac{\|\vec{r}' \times \vec{r}''\|}{\|r'\|} = \|\vec{a} \times \vec{T}\|$ | A vector orthogonal to the unit vector $x(t)\mathbf{i} + y(t)\mathbf{j}$ : <br> $-y(t)\vec{i} + x(t)\vec{j}$ <br> or $y(t)\vec{i} - x(t)\vec{j}$ |