

Name: KEY Date: \_\_\_\_\_

1. Given the vectors  $\mathbf{u}$  and  $\mathbf{v}$ , find  $\mathbf{u} \times \mathbf{v}$  and  $\mathbf{v} \times \mathbf{v}$ .

$$\mathbf{u} = \langle -8, 6, 2 \rangle, \mathbf{v} = \langle 6, -3, -4 \rangle$$

$$\begin{vmatrix} i & j & k \\ -8 & 6 & 2 \\ 6 & -3 & -4 \end{vmatrix} = \langle -24+6 \rangle \mathbf{i} + \langle 12-32 \rangle \mathbf{j} + \langle 24-36 \rangle \mathbf{k} \\ = -18\mathbf{i} - 20\mathbf{j} - 12\mathbf{k}$$

$$\mathbf{v} \times \mathbf{v} = \langle 0, 0, 0 \rangle$$

2. Given the vectors  $\mathbf{u}$  and  $\mathbf{v}$ , find the cross product and determine whether it is orthogonal to both  $\mathbf{u}$  and  $\mathbf{v}$ .

$$\mathbf{u} = \langle -1, 8, 2 \rangle, \mathbf{v} = \langle 4, 10, 5 \rangle$$

$$\begin{vmatrix} i & j & k \\ -1 & 8 & 2 \\ 4 & 10 & 5 \end{vmatrix} = \langle 40-20 \rangle \mathbf{i} + \langle 8+5 \rangle \mathbf{j} + \langle -10-32 \rangle \mathbf{k} \\ = \langle -20, 13, -42 \rangle$$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = \langle -20, 13, -42 \rangle \cdot \langle -1, 8, 2 \rangle \\ = -20 + 8 \cdot 13 - 84 = 0$$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = \langle -20, 13, -42 \rangle \cdot \langle 4, 10, 5 \rangle \\ = -80 + 130 - 210 = 0$$

3. Find the area of a parallelogram that has the given vectors as adjacent sides.

$$\text{area} = \|\mathbf{u} \times \mathbf{v}\|$$

$$\mathbf{u} = \langle -2, 5, 2 \rangle, \mathbf{v} = \langle 6, 2, 3 \rangle$$

$$\begin{vmatrix} i & j & k \\ -2 & 5 & 2 \\ 6 & 2 & 3 \end{vmatrix} = \|\langle 15-4, 12+6, -4-30 \rangle\| \\ = \sqrt{11^2 + 18^2 + 34^2} \\ = \sqrt{121 + 324 + 1156} = \sqrt{1601}$$

4. Find the triple scalar product of the vectors

$$\mathbf{u} = \langle -6, 7, 5 \rangle, \mathbf{v} = \langle 5, 6, -3 \rangle, \mathbf{w} = \langle -4, 0, -7 \rangle$$

 $\mathbf{v} \times \mathbf{w}$ 

$$\begin{vmatrix} i & j & k \\ 5 & 6 & -3 \\ -4 & 0 & -7 \end{vmatrix} = \langle -42, 12+35, 24 \rangle \\ = \langle -42, 47, 24 \rangle$$

$$\mathbf{u} \cdot \mathbf{v} \times \mathbf{w} = \langle -6, 7, 5 \rangle \cdot \langle -42, 47, 24 \rangle$$

$$252 + 329 + 120 = \boxed{701}$$

5. Use the triple scalar product to find the volume of the parallelepiped having adjacent edges given by the vectors

$$\mathbf{u} = \langle 3, 7, 2 \rangle, \mathbf{v} = \langle 0, 9, 4 \rangle, \mathbf{w} = \langle 2, 8, -2 \rangle$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 7 & 2 \\ 0 & 9 & 4 \end{vmatrix} = \langle 28-18, -12, 27 \rangle = \langle 10, -12, 27 \rangle$$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \langle 10, -12, 27 \rangle \cdot \langle 2, 8, -2 \rangle = |20-96-54| = |-130| = 130$$

6. Find a set of parametric equations of the line through the point  $(-6, 9, 4)$  parallel to the vector  $\mathbf{v} = \langle 6, 8, 2 \rangle$ .

$$\langle x, y, z \rangle = \langle -6, 9, 4 \rangle + t \langle 6, 8, 2 \rangle$$

$$x = -6 + 6t$$

$$y = 9 + 8t$$

$$z = 4 + 2t$$

7. Find a set of symmetric equations of the line through the point  $(7, 9, 4)$  parallel to the vector  $\mathbf{v} = \langle 6, 6, 8 \rangle$ .

$$\langle x, y, z \rangle = \langle 7, 9, 4 \rangle + t \langle 6, 6, 8 \rangle$$

$$x = 7 + 6t \Rightarrow t = \frac{x-7}{6}$$

$$y = 9 + 6t \Rightarrow t = \frac{y-9}{6}$$

$$z = 4 + 8t \Rightarrow t = \frac{z-4}{8}$$

$$\frac{x-7}{6} = \frac{y-9}{6} = \frac{z-4}{8}$$

8. Find a set of parametric equations of the line through the points  $(-7, 6, 4)$  and  $(-17, 2, -10)$ .

$$\text{direction vector} = \langle -7, 6, 4 \rangle - \langle -17, 2, -10 \rangle = \langle 10, 4, 14 \rangle$$

$$\langle x, y, z \rangle = \langle -7, 6, 4 \rangle + t \langle 10, 4, 14 \rangle$$

$$x = -7 + 10t$$

$$y = 6 + 4t$$

$$z = 4 + 14t$$

9. Find a set of symmetric equations of the line through the points  $(8, 5, 4)$  and  $(1, 3, -2)$ .

$$\text{direction vector} = \langle 8, 5, 4 \rangle - \langle 1, 3, -2 \rangle = \langle 7, 2, 6 \rangle$$

$$\langle x, y, z \rangle = \langle 8, 5, 4 \rangle + t \langle 7, 2, 6 \rangle$$

$$t = \frac{x-8}{7} \Rightarrow \frac{x-8}{7} = \frac{y-5}{2} = \frac{z-4}{6}$$

$$y = 5 + 2t$$

$$z = 4 + 6t$$

10. Find the set of parametric equations of the line through the point  $(-8, 8, 3)$  and is parallel to the line  $x = 2 + 8t$ ,  $y = 9 + 8t$ , and  $z = 2 + 6t$ .

$$\text{direction vector} = \langle 8, 8, 6 \rangle$$

$$\text{or } \langle 4, 4, 3 \rangle$$

$$\langle x, y, z \rangle = \langle -8, 8, 3 \rangle + t \langle 4, 4, 3 \rangle$$

$$x = -8 + 4t$$

$$y = 8 + 4t$$

$$z = 3 + 3t$$

11. Determine whether any of the lines given below are parallel or identical.

- |   |   |
|---|---|
| $L_1: x = -7 - 4t, y = 3 - 8t, z = -4 - 7t$   | direction vector<br>$\langle -4, -8, -7 \rangle = (-1) \langle 4, 8, 7 \rangle$ |
| $L_2: x = 1 + 8t, y = 19 + 16t, z = 10 + 14t$ | $\langle 8, 16, 14 \rangle = 2 \langle 4, 8, 7 \rangle$                         |
| $L_3: x = 4t, y = 2 - 8t, z = 1 - 7t$         | $\langle 4, -8, -7 \rangle \neq c \langle 4, 8, 7 \rangle$                      |
| $L_4: x = 1 - 8t, y = 19 - 16t, z = 10 - 14t$ | $\langle -8, -16, -14 \rangle = -2 \langle 4, 8, 7 \rangle$                     |

So  $L_1$ ,  $L_2$ , and  $L_4$  are parallel

when  $t = 0$  both  $L_2$  and  $L_4$  contain  $(1, 19, 10)$  so they are the same line.

when  $t = -2$ ,  $L_1$  contain  $(1, 19, 10)$  also. So  $L_1$ ,  $L_2$  and  $L_4$  are the same line.

12. Determine whether any of the lines given below are parallel or identical.

- |   |   |
|---|---|
| $L_1: \frac{x-4}{2} = \frac{y-4}{8} = \frac{z-7}{4}$      | direction vector<br>$\langle 2, 8, 4 \rangle = 2 \langle 1, 4, 2 \rangle$ |
| $L_2: \frac{x-1}{-6} = \frac{y-7}{-24} = \frac{z-10}{12}$ | $\langle -6, -24, 12 \rangle = -6 \langle 1, 4, -2 \rangle$               |
| $L_3: \frac{x}{2} = \frac{y-2}{-8} = \frac{z-1}{-4}$      | $\langle 2, -8, -4 \rangle = 2 \langle 1, -4, -2 \rangle$                 |
| $L_4: \frac{x-1}{6} = \frac{y-7}{24} = \frac{z-10}{-12}$  | $\langle 6, 24, -12 \rangle = 6 \langle 1, 4, -2 \rangle$                 |

parallel

Note that  $L_2$  and  $L_4$  both contain  $(1, 7, 10)$  and are parallel  
So they are the same line.

13. Determine whether the lines given below meet, and, if so, where.

$$x = -8 + 7t, y = 8 + 4t, z = -3 + 2t$$

$$x = 2 + 3s, y = 14 + 2s, z = 2 + 3s$$

At the same point

$$\begin{aligned} -8 + 7t &= 2 + 3s & 7t - 3s &= 10 \\ 8 + 4t &= 14 + 2s \Rightarrow & 4t - 2s &= 6 \\ -3 + 2t &= 2 + 3s & 2t - 3s &= 5 \end{aligned}$$

solve eqn 2 for  $s = 2t + 3$  then plug into eqn 1 & 3

$$\begin{aligned} 7t + 3(2t + 3) &= 10 \Rightarrow t + 9 = 10 \Rightarrow t = 1 \Rightarrow s = 2 \cdot 1 - 3 = -1 \\ 2t - 3(2t + 3) &= 5 \Rightarrow -4t + 9 = 5 \Rightarrow t = 1 \end{aligned}$$

so the lines meet at

$$(-8 + 7 \cdot 1, 8 + 4 \cdot 1, -3 + 2 \cdot 1) = (-1, 12, -1) = (2 + 3(-1), 14 + 2(-1), 2 + 3(-1))$$

14. Determine whether the lines given below are parallel or where they meet.

$$\frac{x-7}{4} = \frac{y-8}{2} = \frac{z-0}{8} \quad \text{direction vector } \langle 4, 2, 8 \rangle \quad \text{not parallel}$$

$$\frac{x-5}{2} = \frac{y-19}{-3} = \frac{z-2}{2} \quad \langle 2, -3, 2 \rangle$$

In parametric form:

$$\begin{aligned} L_1: \quad & x = 4t+7, \quad y = 2t+8, \quad z = 8t \\ L_2: \quad & x = 2s+5, \quad y = -3s+19, \quad z = 2s+2 \end{aligned}$$

$$\begin{aligned} & \Rightarrow \text{at any intersection} \\ & \begin{cases} 4t+7 = 2s+5 \\ 2t+8 = -3s+19 \\ 8t = 2s+2 \end{cases} \Rightarrow \begin{cases} 4t = 2s-2 \\ 4t = -6s+22 \\ 4t = s+1 \end{cases} \end{aligned}$$

$$\begin{aligned} & 2s-2 = s+1 \Rightarrow s=3 \Rightarrow t=1 \\ & \text{so intersection test} \\ & L_1(t=1): (4+7, 2+8, 8) = (11, 10, 8) \quad \text{THEY} \\ & L_2(s=3): (6+5, -9+19, 6+2) = (11, 10, 8) \quad \text{MEET!} \end{aligned}$$

15. Find an equation of a plane passing through the point given and perpendicular to the given vector.

Point: (1, 6, 6) Vector  $\mathbf{v} = \langle 3, 6, 3 \rangle$

$$\underbrace{(\langle x, y, z \rangle - \langle 1, 6, 6 \rangle)}_{\text{vector in plane}} \cdot \underbrace{\langle 3, 6, 3 \rangle}_{\text{normal vector}} = 0 \Rightarrow 3(x-1) + 6(y-6) + 3(z-6) = 0$$

16. Find an equation of a plane passing through the following three points.

$$(-3, -1, -13), (5, 3, 3), (-2, 0, -12)$$

$$\begin{aligned} \text{normal vector: } \mathbf{n} &= \langle -3, -1, -13 \rangle \times \langle 5, 3, 3 \rangle \\ &= \langle 8, 4, 16 \rangle \times \langle 7, 3, 15 \rangle = \begin{vmatrix} i & j & k \\ 8 & 4 & 16 \\ 7 & 3 & 15 \end{vmatrix} \end{aligned}$$

$$\Rightarrow \langle 60-48, 112-120, 24-28 \rangle$$

$$= \langle 12, -8, -4 \rangle$$

or equivalently

$$\vec{n} = \langle 3, -2, -1 \rangle$$

$$\text{Equation } 3(x-5) - 2(y-3) - (z-3) = 0$$

17. Find an equation of a plane passing through the points

$$(-1, 1, 1), (1, 2, 6)$$

and perpendicular to the plane

$$2x+y+5z+7=0$$

The normal vector for the new plane must be orthogonal to both

$$\langle -1, 1, 1 \rangle \times \langle 1, 2, 6 \rangle = \langle 2, 1, 5 \rangle$$

and (orthogonal to) the normal vector of  $2x+y+5z+7=0$ ,

also  $\langle 2, 1, 5 \rangle$ .

So let  $\vec{n} = \langle -1, 2, 0 \rangle$

(or  $\langle 5, 0, -2 \rangle$  or  $\langle 0, 5, -1 \rangle$  or

etc.)

Eqn:

$$-(x+1) + 2(y-1) + 0(z-1) = 0$$

$$\equiv 2y - x = 3$$

18. Determine whether the following planes are parallel, orthogonal, or neither. If they are neither parallel nor orthogonal, find the angle of intersection.

$$-4x - 0y + 4z + 4 = 0 \quad \text{normal vectors} \\ \langle -4, 0, 4 \rangle = \vec{n}_1$$

$$2x + y + 2z - 4 = 0 \quad \langle 2, 1, 2 \rangle = \vec{n}_2$$

$$\vec{n}_1 \cdot \vec{n}_2 = -8 + 0 + 8 = 0$$

$\Rightarrow$  planes are orthogonal

19. Determine whether the following planes are parallel, orthogonal, or neither. If they are neither parallel nor orthogonal, find the angle of intersection.

$$-2x - 7y + 4z + 2 = 0 \quad \text{normal vectors} \\ \langle -2, -7, 4 \rangle = \vec{n}_1$$

$$2x + 8y - 8z - 4 = 0 \quad \langle 2, 8, -8 \rangle = \vec{n}_2$$

$$\vec{n}_1 \cdot \vec{n}_2 = -92 \Rightarrow \text{not orthogonal}$$

$$\vec{n}_1 \neq c\vec{n}_2 \text{ for any } c \in \mathbb{R} \Rightarrow \text{not parallel}$$

$$\cos \theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{\|\vec{n}_1\| \|\vec{n}_2\|} = \frac{-92}{\sqrt{4+49+16} \sqrt{4+64+64}} \\ = \frac{-92}{\sqrt{69} \sqrt{132}} \\ \Rightarrow \theta = \arccos(-.964) \\ = .269 \text{ radians} \\ = 15.42^\circ$$

20. Find the distance between the point  $(1, 2, 3)$  and the plane given below.

$$\vec{n} = \langle 5, -8, 7 \rangle$$

$$\text{Point in the plane} \\ (0, 0, 2)$$

$$\text{vector to plane: } \overrightarrow{(1,2,3)(0,0,2)} = \langle 1, 2, 1 \rangle = \vec{v}$$

$$\text{distance to plane} \\ \left| \frac{\vec{v} \cdot \vec{n}}{\|\vec{n}\|} \right| = \frac{|\langle 1, 2, 1 \rangle \cdot \langle 5, -8, 7 \rangle|}{\sqrt{25+64+49}} = \frac{4}{\sqrt{138}} \approx .3405$$

21. Find the distance between the planes given below.

$$P_1: 7x - 4y + 2z - 5 = 0 \quad \text{normal vector} \quad \langle 7, -4, 2 \rangle = \vec{n} \quad \text{point in Plane} \\ (1, 0, -1)$$

$$P_2: 14x - 8y + 4z - 16 = 0 \quad " \quad (0, -2, 0)$$

$$\text{vector between planes: } \overrightarrow{(1, 0, -1)(0, -2, 0)} = \langle 1, 2, -1 \rangle = \vec{v}$$

distance between planes:

$$\left| \frac{\vec{v} \cdot \vec{n}}{\|\vec{n}\|} \right| = \frac{|\langle 1, 2, -1 \rangle \cdot \langle 7, -4, 2 \rangle|}{\sqrt{49+16+4}} = \frac{|7-8-2|}{\sqrt{69}} = \frac{3}{\sqrt{69}} \approx .3612$$

22. Identify the following quadratic surface.

$$\frac{x^2}{3} + \frac{y^2}{14} + \frac{z^2}{4} = 1$$

intersecting coordinate planes:

$$x=0: \frac{y^2}{14} + \frac{z^2}{4} = 1: \text{ellipse}$$

$$y=0: \frac{x^2}{3} + \frac{z^2}{4} = 1: \text{ellipse}$$

$$z=0: \frac{x^2}{3} + \frac{y^2}{14} = 1 \quad \Rightarrow \text{ellipsoid}$$

23. Identify the following quadratic surface.

$$\frac{x^2}{4} + \frac{y^2}{8} - \frac{z^2}{16} = 1$$

intersecting coordinate planes

$$x=0: \frac{y^2}{8} - \frac{z^2}{16} = 1: \text{hyperbola}$$

$$y=0: \frac{x^2}{4} - \frac{z^2}{16} = 1: \text{hyperbola}$$

$$z=0: \frac{x^2}{4} + \frac{y^2}{8} = 1: \text{ellipse} \quad \Rightarrow$$

hyperboloid  
of one  
sheet

24. Identify the following quadratic surface.

$$\frac{x^2}{4} - \frac{y^2}{6} - \frac{z^2}{12} = 1$$

intersecting planes

$$x=0: -\frac{y^2}{6} - \frac{z^2}{12} = 1 \quad \text{none}$$

$$y=0: \frac{x^2}{4} - \frac{z^2}{12} = 1 \quad \text{hyperbola}$$

$$z=0: \frac{x^2}{4} - \frac{y^2}{6} = 1 \quad \text{hyperbola}$$

hyperboloid  
of two  
sheets

25. Identify the following quadratic surface.

$$\frac{x^2}{2} + \frac{y^2}{16} - \frac{z^2}{10} = 0$$

intersecting planes

$$x=0: \frac{y^2}{16} - \frac{z^2}{10} = 0 \quad \text{crossed lines}$$

$$y=0: \frac{x^2}{2} - \frac{z^2}{10} = 0 \quad \text{crossed lines}$$

$$z=0: \frac{x^2}{2} + \frac{y^2}{16} = 0 \quad \text{point}$$

elliptic  
cone

26. Identify the following quadratic surface.

$$z = \frac{x^2}{4} + \frac{y^2}{4}$$

intersecting planes

$$x=0: z = \frac{y^2}{4} \quad \text{parabola}$$

$$y=0: z = \frac{x^2}{4} \quad \text{parabola}$$

$$z=1: 1 = \frac{x^2}{4} + \frac{y^2}{4} \quad \text{circle}$$

elliptic  
paraboloid

27. Identify the following quadratic surface.

$$z = \frac{x^2}{8} - \frac{y^2}{4}$$

intersecting planes

$x=0 : z = -\frac{y^2}{4}$  parabola

$y=0 : z = \frac{x^2}{8}$  parabola

$z=1 : 1 = \frac{x^2}{8} - \frac{y^2}{4}$  hyperbola

$\Rightarrow$  hyperbolic paraboloid

28. Find an equation of the surface of revolution generated by revolving the curve given below in the indicated coordinate plane about the given axis.

Equation of Curve  
 $z^2 = \sqrt{7}y$

Coordinate Plane  
 yz-plane

Axis of Revolution  
 y-axis

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29. Find an equation of the surface of revolution generated by revolving the curve given below in the indicated coordinate plane about the given axis.

Equation of Curve  
 $z = 6x$

Coordinate Plane  
 xz-plane

Axis of Revolution  
 z-axis

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30. Find an equation of a generating curve given the equation of its surface of revolution is

$$x^2 + y^2 = \frac{z^2}{36}$$

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Equation of Curve

Coordinate Plane

Axis of Revolution

31. Find the equation of the surface satisfying the conditions, and identify the surface.

The set of points equidistant from the point  $(-2, 3, 4)$  and the plane  $y=4$ .

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32. Convert the following point from cylindrical coordinates to rectangular coordinates.

$$\left(8, \frac{\pi}{6}, 6\right) \quad x = r\cos\theta = 8\cos\left(\frac{\pi}{6}\right) = 8 \cdot \frac{\sqrt{3}}{2} = 4\sqrt{3}$$

$$y = r\sin\theta = 8\sin\left(\frac{\pi}{6}\right) = 8 \cdot \frac{1}{2} = 4$$

$$z = z = 6$$

$$(4\sqrt{3}, 4, 6)$$

33. Convert the following point from rectangular coordinates to cylindrical coordinates. Give any angles in radians.

$$(4, 1, 4) \quad r = \sqrt{x^2 + y^2} = \sqrt{16 + 1} = \sqrt{17}$$

$$\tan\theta = \frac{1}{4} \Rightarrow \theta = .245 \text{ radians}$$

$$z = 4 \quad (\sqrt{17}, .245, 4)$$

34. Find an equation in cylindrical coordinates for the equation given in rectangular coordinates.

$$z = 49x^2 + 49y^2 - 4$$

$$z = 49(x^2 + y^2) - 4$$

$$z = 49r^2 - 4$$

35. Find an equation in cylindrical coordinates for the equation given in rectangular coordinates.

$$9x^2 + 9y^2 = 2x$$

$$9(x^2 + y^2) = 2x$$

$$9r^2 = 2r\cos\theta$$

$$r = \frac{2}{9}\cos\theta$$

36. Find an equation in cylindrical coordinates for the equation given in rectangular coordinates.

$$25x^2 + 25y^2 - 4z^2 = g$$

$$25(x^2+y^2) - 4z^2 = g$$

$$25r^2 - 4z^2 = g$$

$$z^2 = \frac{25r^2 - g}{4}$$

37. Find an equation in rectangular coordinates for the equation given in cylindrical coordinates.

$$r = 5\sin\theta$$

$$r^2 = 25r\sin\theta$$

$$x^2 + y^2 = 25y$$

38. Find an equation in rectangular coordinates for the equation given in cylindrical coordinates

$$r = 4z$$

$$r^2 = 16z^2$$

$$x^2 + y^2 = 16z^2$$

39. Find an equation in rectangular coordinates for the equation given in cylindrical coordinates.

$$r^2 + z^2 = 25$$

$$x^2 + y^2 + z^2 = 25$$

40. Convert the point from spherical coordinates to rectangular coordinates.

$$\left(4, \frac{\pi}{6}, \frac{\pi}{16}\right)$$

$$z = \rho \cos\phi = 4 \cos \frac{\pi}{16} = 3.923$$

$$r = \rho \sin\phi = 4 \sin \frac{\pi}{16} = .7804$$

$$x = r \cos\theta = 4 \sin \frac{\pi}{16} \cos \frac{\pi}{6} = 2\sqrt{3} \sin \frac{\pi}{16} = .6758$$

$$y = r \sin\theta = 4 \sin \frac{\pi}{16} \sin \frac{\pi}{6} = 2 \sin \frac{\pi}{16} = .3902$$

$$(.6758, .3902, 3.923)$$

41. Find an equation in spherical coordinates for the equation given in rectangular coordinates.

$$\begin{aligned} y &= 2 \\ r \sin \theta &= 2 \\ \rho \sin \phi \sin \theta &= 2 \end{aligned}$$

solve for  $\rho$

$$\rho = 2 \csc \phi \csc \theta$$

42. Find an equation in spherical coordinates for the equation given in rectangular coordinates.

$$\begin{aligned} x^2 + y^2 - 6z^2 &= 3 \\ r^2 - 6z^2 &= 3 \\ (\rho \sin \phi)^2 - 6(\rho \cos \phi)^2 &= 3 \end{aligned}$$

solve for  $\rho^2$

$$\rho^2 [\sin^2 \phi - 6 \cos^2 \phi] = 3$$

$$\rho^2 = \frac{3}{\sin^2 \phi - 6 \cos^2 \phi}$$

43. Find an equation in rectangular coordinates for the equation given in spherical coordinates.

$$\theta = \frac{\pi}{8} \quad \tan\left(\frac{\pi}{8}\right) = \frac{y}{x} \Rightarrow y = x \tan\left(\frac{\pi}{8}\right) \text{ or } y = .414x$$

The coefficients below are given to two decimal places.

44. Find an equation in rectangular coordinates for the equation given in spherical coordinates.

$$\begin{aligned} \rho &= 3 \csc \phi \csc \theta \\ \rho &= \frac{3}{\sin \phi \sin \theta} \end{aligned}$$

solve for  $\rho \sin \phi \sin \theta = 3$

$$\begin{aligned} r \sin \theta &= 3 \\ y &= 3 \end{aligned}$$

45. Convert the following point from cylindrical coordinates to spherical coordinates.

$$\begin{aligned} \left(6, \frac{\pi}{3}, 8\right) &\quad \tan \phi = \frac{r}{z} = \frac{6}{8} = \frac{3}{4} \\ (r, \theta, z) &\quad \phi = \arctan\left(\frac{3}{4}\right) = .6435 \text{ radians} \\ \theta &= \frac{\pi}{3} \\ r^2 &= r^2 + z^2 \\ r^2 &= 36 + 64 \\ r &= 10 \end{aligned}$$

$$(\rho, \theta, \phi) = \left(10, \frac{\pi}{3}, .6435\right)$$

46. Convert the following point from spherical coordinates to cylindrical coordinates.

$$\left(7, \frac{\pi}{16}, \frac{\pi}{4}\right) \quad \begin{aligned} z &= \rho \cos \phi = 7 \cos\left(\frac{\pi}{4}\right) = \frac{7}{\sqrt{2}} \\ r &= \rho \sin \phi = 7 \sin\left(\frac{\pi}{4}\right) = \frac{7}{\sqrt{2}} \\ \theta &= \theta = \frac{\pi}{16} \end{aligned}$$

$$\left(\frac{7}{\sqrt{2}}, \frac{\pi}{16}, \frac{7}{\sqrt{2}}\right) = (r, \theta, z)$$

47. Identify the solid that has the given description in spherical coordinates.

$$\rho = 1, 0 \leq \theta \leq \frac{\pi}{2}, \frac{\pi}{4} \leq \phi \leq \frac{\pi}{2}$$

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