

# Practice Problems 8 and 9

To find the intersection of two curves in polar coordinates:

$r$  of curve 1 =  $r$  of curve 2 for the same  $\theta$ .

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8. Find the points of intersection of the graphs of the equations.

$$r = 1 + \cos \theta$$

$$r = 3 \cos \theta$$

$$1 + \cos \theta = 3 \cos \theta$$

$$1 = 2 \cos \theta$$

$$\frac{1}{2} = \cos \theta$$

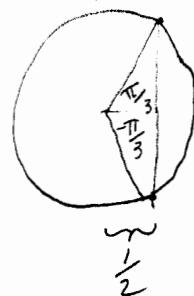
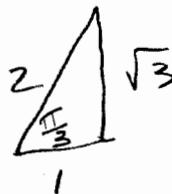
$$\theta = \frac{\pi}{3} + 2n\pi$$

$$\text{or } -\frac{\pi}{3} + 2n\pi$$

$$r = 1 + \cos \theta = 1 + \frac{1}{2} = \frac{3}{2}$$

intersections at

$$\left(\frac{3}{2}, \frac{\pi}{3}\right) \text{ and } \left(\frac{3}{2}, -\frac{\pi}{3}\right)$$



9. Find the points of intersection of the graphs of the equations.

$$r = \frac{\theta}{1.9}$$
$$r = 1.9$$

For first solution

$$\frac{\theta}{1.9} = 1.9 \Rightarrow \theta = 1.9^2 = 3.61$$
$$(1.9, 3.61)$$

The second curve is also given by

$r = -1.9$   
so another solution is

$$\frac{\theta}{1.9} = -1.9 \Rightarrow \theta = -1.9^2 = -3.61$$
$$(-1.9, -3.61)$$

# Practice Problems 10-12

Arclength in polar coordinates for  $r=f(\theta)$

from pg 698:

$$s = \int_{\alpha}^{\beta} \sqrt{(f(\theta))^2 + (f'(\theta))^2} d\theta$$



10. Find the length of the curve over the given interval.

$$r = 6 \cos \theta, -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$f(\theta) = 6 \cos \theta \quad f'(\theta) = -6 \sin \theta$$

$$s = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{36 \cos^2 \theta + 36 \sin^2 \theta} d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 6 d\theta = \left[ 6\theta \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 6\pi$$

11. Find the length of the curve over the given interval.

$$r = 7 + 7 \sin \theta, 0 \leq \theta \leq 2\pi$$

$$s = \int_0^{2\pi} \sqrt{(7+7\sin\theta)^2 + (7\cos\theta)^2} d\theta$$

$$= 7\sqrt{2} \int_0^{2\pi} \sqrt{1+\sin\theta} d\theta$$

remember cosine is sine of the complement

$$\text{so } 1+\sin\theta = 1+\cos(\frac{\pi}{2}-\theta)$$

$$= 2\cos^2\left(\frac{\frac{\pi}{2}-\theta}{2}\right)$$

by  $\frac{1}{2}$  angle formula

$$s = 7\sqrt{2} \int_0^{2\pi} \sqrt{2\cos^2\left(\frac{\frac{\pi}{2}-\theta}{2}\right)} d\theta$$

$$= 14 \int_0^{2\pi} |\cos(\frac{\frac{\pi}{2}-\theta}{2})| d\theta$$

$$u = \frac{\frac{\pi}{2}-\theta}{2}$$

$$du = -\frac{d\theta}{2}$$

$$-2du = d\theta$$

$$s = -28 \int_{\frac{\pi}{4}}^{-\frac{3\pi}{4}} |\cos u| du$$

$$S = -28 \int_{\frac{\pi}{4}}^{-\frac{\pi}{2}} \cos u du + 28 \int_{-\frac{\pi}{2}}^{-\frac{3\pi}{4}} \cos u du$$

$$= 28 \left[ \int_{-\frac{\pi}{2}}^{\frac{\pi}{4}} \cos u du - \int_{-\frac{3\pi}{4}}^{-\frac{\pi}{2}} \cos u du \right]$$

$$= 28 \left[ [\sin u]_{-\frac{\pi}{2}}^{\frac{\pi}{4}} - [\sin u]_{-\frac{3\pi}{4}}^{-\frac{\pi}{2}} \right]$$

$$= 28 \left[ \left( \frac{1}{\sqrt{2}} - -1 \right) - \left( -1 - -\frac{1}{\sqrt{2}} \right) \right]$$

$$= 28 \cdot 2 = 56$$

12. Find the length of the curve over the given interval.

$$r = 3(1 + \cos \theta), 0 \leq \theta \leq 2\pi$$

$$f(\theta) = 3(1 + \cos \theta) \quad f'(\theta) = -3 \sin \theta$$

$$\begin{aligned} s &= \int_0^{2\pi} \sqrt{3^2(1 + \cos \theta)^2 + 3^2 \sin^2 \theta} d\theta \\ &= 3\sqrt{2} \int_0^{2\pi} \sqrt{1 + \cos \theta} d\theta \end{aligned}$$

From the double-angle formula

$$\cos(2u) = 2\cos^2 u - 1$$

we have

$$1 + \cos \theta = 2 \cos^2 \frac{\theta}{2}$$

$$s = 6 \int_0^{2\pi} |\cos(\frac{\theta}{2})| d\theta$$

with  $u = \frac{\theta}{2}$        $2du = d\theta$

$$\begin{aligned} s &= 12 \int_0^{\pi} |\cos u| du = 12 \left[ \int_0^{\frac{\pi}{2}} \cos u du - \int_{\frac{\pi}{2}}^{\pi} \cos u du \right] \\ &= 12 \left( [\sin u]_0^{\frac{\pi}{2}} - [\sin u]_{\frac{\pi}{2}}^{\pi} \right) \\ &= 12 (1 - 0 - (0 - 1)) = 24 \end{aligned}$$