M252 Practice Questions for Chapter **15**
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1. Find the gradient vector for the scalar function. (That is, find the conservative vector field for the potential function.)

$f(x,y)=7x^2+8xy+2y^2$ $\nabla f=\langle 14x+8y, 8x+4y\rangle$
or $(14x+8y)\vec{i}+(8x+4y)\vec{j}$

2. Find the gradient vector for the scalar function. (That is, find the conservative vector field for the potential function.)

$\nabla f=\langle 9\cos 9x\cos 3y, -3\sin 9x\sin 3y\rangle$

$f(x,y)=\sin 9x\cos 3y$

3. Determine whether the vector field is conservative.

$N=-9xy^2$ $M=63y^3$

$\vec{\mathbf{F}}(x,y)=9y^2\left(7y\hat{\mathbf{i}}-x\hat{\mathbf{j}}\right)$

$\frac{\partial N}{\partial x}=-9y^2$ $\frac{\partial M}{\partial y}=189y^2$

A) Conservative
B) Not Conservative

$\frac{\partial N}{\partial x}\neq\frac{\partial M}{\partial y}$

4. Determine whether the vector field is conservative. If it is, find a potential function for the vector field.

$\frac{\partial\, 7x^6y}{\partial y}=7x^6$ $\frac{\partial\, x^7}{\partial x}=7x^6 \Rightarrow$ conservative

$\vec{\mathbf{F}}(x,y)=7x^6y\hat{\mathbf{i}}+x^7\hat{\mathbf{j}}$

$f(x,y)=x^7y+c$

5. Determine whether the vector field is conservative. If it is, find a potential function for the vector field.

$\frac{\partial \frac{6y}{x}}{\partial y}=\frac{6}{x}$ $\frac{\partial\, -\frac{x^6}{y^6}}{\partial x}=-\frac{6x^5}{y^6}\Rightarrow$ NOT conservative

$\vec{\mathbf{F}}(x,y)=\frac{6y}{x}\hat{\mathbf{i}}-\frac{x^6}{y^6}\hat{\mathbf{j}}$

6. Find the curl for the vector field at the given point.

$\vec{\mathbf{F}}(x,y,z)=2xyz\hat{\mathbf{i}}+2y\hat{\mathbf{j}}+2z\hat{\mathbf{k}},\quad (2,3,2)$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz & 2y & 2z \end{vmatrix} = 0\vec{i}+2xy\vec{j}-2xz\vec{k}$$

at $(2,3,2)$ we have
curl $F=\langle 0,12,-8\rangle$

7. Determine whether the vector field is conservative. If it is, find a potential function for the vector field.

$\frac{\partial M}{\partial y} = 12x^2y^3z^5 = \frac{\partial N}{\partial x}$, $\frac{\partial M}{\partial z} = 15x^2y^4z^4 = \frac{\partial P}{\partial x}$,

$\vec{\mathbf{F}}(x,y,z) = 3x^2y^4z^5\hat{\mathbf{i}} + 4x^3y^3z^5\hat{\mathbf{j}} + 5x^3y^4z^4\hat{\mathbf{k}}$

$\frac{\partial N}{\partial z} = 20x^3y^3z^4 = \frac{\partial P}{\partial y}$

$f(x,y,z) = x^3y^4z^5$

8. Find the divergence of the vector field.

$\vec{\mathbf{F}}(x,y,z) = 9x^4\hat{\mathbf{i}} - xy^3\hat{\mathbf{j}}$

$$\frac{\partial 9x^4}{\partial x} + \frac{\partial -xy^3}{\partial y} = 36x^3 - 3xy^2$$

9. Find the divergence of the vector field at the given point.

$\vec{\mathbf{F}}(x,y,z) = 8xyz\hat{\mathbf{i}} + 8y\hat{\mathbf{j}} + 8x\hat{\mathbf{k}}$, $(8,9,8)$

$$\frac{\partial 8xyz}{\partial x} + \frac{\partial 8y}{\partial y} + \frac{\partial 8x}{\partial z} = 8yz + 8$$

at $(8,9,8)$, $\nabla \cdot F = 8 \cdot 9 \cdot 8 + 8 = 584$

12. Evaluate the line integral along the given path.

$\int_C (2x - 7y)\,ds \quad C: \vec{\mathbf{r}}(t) = 7t\hat{\mathbf{i}} + 3t\hat{\mathbf{j}}, \quad 0 \le t \le 8$

$r'(t) = \langle 7, 3 \rangle \quad \|r'(t)\| = \sqrt{49+9} = \sqrt{58} \quad ds = \|r'(t)\|\,dt$

$= \sqrt{58}\,dt$

$$\int_C (2x - 7y)\,ds = \int_0^8 \left(2(7t) - 7(3t)\right)\sqrt{58}\,dt$$

$$\sqrt{58}\int_0^8 -7t\,dt = \sqrt{58}\left(-7 \cdot \frac{8^2}{2}\right) = -224\sqrt{58}$$

13. Evaluate $\int_C (x^2+y^2)\,ds$ where the path C is:

(i) the x-axis from $x=0$ to $x=1$; $x=t \quad y=0 \quad r(t)=\langle t,0\rangle \quad r'(t)=\langle 1,0\rangle$

$ds = \|r'(t)\|dt = dt$ $\quad \int_0^1 t^2+0^2\,dt = \left[\frac{t^3}{3}\right]_0^1 = \frac{1}{3}$

(ii) the y-axis from $y=1$ to $y=3$. $x=0,\ y=t,\ 1\le t\le 3,\ \|r'(t)\|=1 \quad ds=dt$

$\int_1^3 0^2+t^2\,dt = \left[\frac{t^3}{3}\right]_1^3 = 9-\frac{1}{3} = \frac{26}{3}$

14. Evaluate $\int_C (x+49\sqrt{y})\,ds$ where the path C is:

(i) the line from (0,0) to (1,1); $r(t)=\langle t^2,t^2\rangle,\ 0\le t\le 1 \quad r'(t)=\langle 2t,2t\rangle,\ ds=2\sqrt{2}\,t\,dt$

$\int_0^1 (t^2+49t)2\sqrt{2}\,t\,dt = 2\sqrt{2}\int_0^1 t^3+49t^2\,dt = 2\sqrt{2}\left[\frac{t^4}{4}+\frac{49t^3}{3}\right]_0^1 = 2\sqrt{2}\left(\frac{1}{4}+\frac{49}{3}\right)$

$= 199\sqrt{2}/6$

(ii) the line from (0,0) to (3,9). $r(t)=\langle 3t^2, 9t^2\rangle \quad 0\le t\le 1 \quad r'(t)=\langle 6t, 18t\rangle$

$ds = 6t\sqrt{10}\,dt$

$\int_0^1 (3t^2+3\cdot 49t)6t\sqrt{10}\,dt = 18\sqrt{10}\int_0^1 t^3+49t^2\,dt = 18\sqrt{10}\left[\frac{t^4}{4}+\frac{49t^3}{3}\right]_0^1 = 18\sqrt{10}\left(\frac{1}{4}+\frac{49}{3}\right)$

$= \frac{597\sqrt{10}}{2}$

15. Evaluate $\int_C \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$ where C is represented by $\hat{\mathbf{r}}(t)$.

$\vec{\mathbf{F}}(x,y) = xy\hat{\mathbf{i}} + y\hat{\mathbf{j}}$

$C: \vec{\mathbf{r}}(t) = 32t\hat{\mathbf{i}} + t\hat{\mathbf{j}}, \quad 0\le t\le 8$

$d\vec{r} = \langle 32, 1\rangle dt$

$\int_0^8 \langle xy, y\rangle\cdot\langle 32,1\rangle dt = \int_0^8 \langle 32t\cdot t, t\rangle\langle 32,1\rangle dt \rightarrow \int_0^8 (32^2t^2+t)dt$

$= \frac{32^2\cdot 8^3}{3} + \frac{8^2}{2}$

$= 174794.\frac{2}{3}$

16. Evaluate $\int_C \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$ where C is represented by $\vec{\mathbf{r}}(t)$.

$\vec{\mathbf{F}}(x,y) = 5x\hat{\mathbf{i}} + 9y\hat{\mathbf{j}}$

$C: \vec{\mathbf{r}}(t) = t\hat{\mathbf{i}} + \sqrt{4-t^2}\,\hat{\mathbf{j}}, \quad -2\le t\le 2$

$d\vec{r} = \langle 1, \frac{-t}{\sqrt{4-t^2}}\rangle dt$

$\int_{-2}^{2} \langle 5t, 9\sqrt{4-t^2}\rangle\cdot\langle 1, \frac{-t}{\sqrt{4-t^2}}\rangle dt$

$= \int_{-2}^{2} (5t-9t)dt = \left[-2t^2\right]_{-2}^{2} = 0$

17. Evaluate the line integral along the path C given by $x=4t$, $y=20t$, where $0\le t\le 1$.

$\int_C (x+4y^2)\,ds$ $\quad \vec{r}(t) = 4t\langle 1,5\rangle \quad 0\le t\le 1 \quad r'(t) = 4\langle 1,5\rangle$

$ds = 4\sqrt{26}\,dt$

$\int_0^1 (4t+4(20t)^2)4\sqrt{26}\,dt = 16\sqrt{26}\int_0^1 t+400t^2\,dt = 16\sqrt{26}\left[\frac{t^2}{2}+\frac{400t^3}{3}\right]_0^1$

$= 16\sqrt{26}\left(\frac{1}{2}+\frac{400}{3}\right) = \frac{6424}{3}\sqrt{26}$

18. Evaluate the line integral

$$\int_C (2x-y)\,dx+(x+3y)\,dy$$

along the path *C*, where *C* is:

(i) the *x*-axis from $x=0$ to $x=8$. $r(t)=\langle t,0\rangle$, $dx=dt$ $dy=0$

$$\int_0^8 (2t-0)\,dt+(t+3\cdot 0)\cdot 0=\left[t^2\right]_0^8=64$$

(ii) the *y*-axis from $y=0$ to $y=12$. $r(t)=\langle 0,t\rangle$, $dx=0$, $dy=dt$

$$\int_0^{12} (2\cdot 0-t)\cdot 0+(0+3t)\,dt=\left[\frac{3t^2}{2}\right]_0^{12}=216$$

19. Find the area of the lateral surface over the curve *C* in the *xy*-plane and under the surface $z=f(x,y)$, where

Lateral surface area $=\int_C f(x,y)\,ds$

$f(x,y)=4$, C: line from (0,0) to (5,6).

$r(t)=\langle 5t,6t\rangle$ $ds=\sqrt{25+36}\,dt$

$$\int_0^1 4\sqrt{61}\,dt=4\sqrt{61}$$

20. Set up and evaluate the integral $\int_C \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$ for each parametric representation of *C*.

$\vec{\mathbf{F}}(x,y)=x^2\hat{\mathbf{i}}+xy\hat{\mathbf{j}}$ on C $\vec{F}=\langle 9t^2,21t^3\rangle$

(i) $\vec{\mathbf{r}}_1(t)=3t\hat{\mathbf{i}}+7t^2\hat{\mathbf{j}}$, $0\le t\le 1$ $d\vec{r}=\langle 3,14t\rangle dt$

$$\int_0^1 3t^2\langle 3,7t\rangle\cdot\langle 3,14t\rangle\,dt$$

$$=3\int_0^1 9t^2+98t^4\,dt$$

$$=3\left[3t^3+\frac{98}{5}t^5\right]_0^1$$

$$=3\left(3+\frac{98}{5}\right)=67.8$$

(ii) $\vec{\mathbf{r}}_2(t)=3\sin\theta\hat{\mathbf{i}}+7\sin^2\theta\hat{\mathbf{j}}$, $0\le\theta\le\frac{\pi}{2}$

on C $\vec{F}=\langle 9\sin^2\theta,21\sin^3\theta\rangle$

$d\vec{r}=\langle 3\cos\theta,14\sin\theta\cos\theta\rangle d\theta$

$$\int_0^{\frac{\pi}{2}} 3\sin^2\theta\cos\theta\langle 3,7\sin\theta\rangle\cdot\langle 3,14\sin\theta\rangle\,d\theta$$

$u=\sin\theta$
$du=\cos\theta\,d\theta$

$$=3\int_0^{\frac{\pi}{2}}\sin^2\theta\cos\theta\left(9+98\sin^2\theta\right)d\theta$$

$$3\int_0^1 u^2(9+98u^2)\,du=3\left[3u^3+\frac{98}{5}u^5\right]_0^1=3\left(3+\frac{98}{5}\right)$$

$$=67.8$$

21. Determine whether or not the vector field is conservative.

$\vec{\mathbf{F}}(x,y) = 40x^7y^7\hat{\mathbf{i}} + 35x^8y^6\hat{\mathbf{j}}$

A) Conservative

B) Not conservative

$\frac{\partial M}{\partial y} = 280x^7y^6 \quad \frac{\partial N}{\partial x} = 280x^7y^6$

22. Find the value of the line integral $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where $\vec{\mathbf{F}}(x,y) = 2xy\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}$.

(i) $\vec{\mathbf{r}}_1(t) = t\hat{\mathbf{i}} + t^3\hat{\mathbf{j}}, \quad 0 \le t \le 1$ $\quad \vec{F} = \langle 2t^4, t^2 \rangle, \ d\vec{r} = \langle 1, 3t^2 \rangle dt$

$$\int_0^1 t^2\langle 2t^2, 1\rangle \cdot \langle 1, 3t^2\rangle dt = \int_0^1 5t^4 dt = \left[t^5\right]_0^1 = 1$$

(ii) $\vec{\mathbf{r}}_2(t) = t\hat{\mathbf{i}} + t^7\hat{\mathbf{j}}, \quad 0 \le t \le 1$ $\quad \vec{F} = \langle 2t^8, t^2 \rangle, \ d\vec{F} = \langle 1, 7t^6 \rangle dt$

$$\int_0^1 t^2\langle 2t^6, 1\rangle \cdot \langle 1, 7t^6\rangle dt = \int_0^1 9t^8 dt = \left[t^9\right]_0^1 = 1$$

23. Find the value of the line integral $\int_C (2x - 5y + 7)dx - (5x + 7y - 8)dy$.

(i) C: the curve $x = \sqrt{49 - y^2}$ from $(0,-7)$ to $(0,7)$

$x = 7\cos\theta, \ y = 7\sin\theta, \ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$dx = -7\sin\theta \, d\theta \quad dy = 7\cos\theta \, d\theta$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2(7\cos\theta) - 5(7\sin\theta) + 7)(-7\sin\theta \, d\theta) - (5(7\cos\theta) + 7(7\sin\theta) - 8)(7\cos\theta \, d\theta)$$

factor out 7 and drop odd terms

$$7\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(35(\sin^2\theta - \cos^2\theta) + 8\cos\theta\right) d\theta = 7\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -35\cos(2\theta) + 8\cos\theta \, d\theta = 7\left[-\frac{35}{2}\sin(2\theta) + 8\sin\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 112$$

(ii) C: from $(0,-7)$ to $(0,7)$ along $x = \sqrt{49 - y^2}$, then back to $(0,-7)$ along the y-axis (a closed curve).

Using Green's Theorem $\int_D (-5) - (-5) \, dA = 0$ / check by integrating along y axis

24. Find the value of the line integral $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where $\vec{\mathbf{F}}(x,y) = 3yz\hat{\mathbf{i}} + 3xz\hat{\mathbf{j}} + 3xy\hat{\mathbf{k}}$.

(i) $\vec{\mathbf{r}}_1(t) = t\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + t\hat{\mathbf{k}}, \quad 0 \le t \le 81$ $\quad \vec{F} = \langle 27t, 3t^2, 27t \rangle, \ d\vec{r} = \langle 1, 0, 1 \rangle dt$

$$\int_0^{81} 3t\langle 9, t, 9\rangle \cdot \langle 1, 0, 1\rangle dt = 54\int_0^{81} t\,dt = \frac{54 \cdot 81^2}{2} = 27 \cdot 81^2 = 3^{11}$$

(ii) $\vec{\mathbf{r}}_2(t) = t^2\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + t^2\hat{\mathbf{k}}, \quad 0 \le t \le 9$ $\quad \vec{F} = \langle 27t^2, 3t^4, 27t^2 \rangle, \ d\vec{r} = \langle 2t, 0, 2t \rangle dt$

$$\int_0^9 3t^2\langle 9, t^2, 9\rangle \cdot \langle 2t, 0, 2t\rangle dt = 108\int_0^9 t^3 dt = \frac{108}{4}9^4 = 27 \cdot 9^4 = 3^{11}$$

25. Evaluate the line integral using the Fundamental Theorem of Line Integrals. Use a computer algebra system to verify your results.

$\int_C (4y\hat{\mathbf{i}}+4x\hat{\mathbf{j}})\cdot d\vec{r}$ $\quad f(x,y)=4xy$ potential function

$\int_C \vec{F}\cdot d\vec{r} = f(9,4)-f(0,0)=144$

C: a smooth curve from (0,0) to (9,4)

26. Evaluate the line integral using the Fundamental Theorem of Line Integrals. Use a computer algebra system to verify your results.

$\int_C \frac{2x}{(x^2+y^2)^2}dx+\frac{2y}{(x^2+y^2)^2}dy$ $\quad f(x,y)=\frac{-1}{x^2+y^2}$ (potential function

$\int_C \vec{F}\cdot d\vec{r} = f(-1,3)-f(13,3) = \frac{-1}{1+9}-\frac{-1}{169+9}=\frac{-42}{445}$

C: circle $(x-6)^2+(y-3)^2=49$ clockwise from (13,3) to (−1,3)

28. Use Green's Theorem to evaluate the integral

$\int_C (y-x)dx+(5x-y)dy = \iint_D 5-1\,dA = 4\int_0^{19} x-(x^2-18x)dx = 4\int_0^{19} 19x-x^2dx =$

for the path C: boundary of the region lying between the graphs of $y=x$ and $y=x^2-18x$

$4\left[\frac{19x^2}{2}-\frac{x^3}{3}\right]_0^{19}$

$\to 4\cdot 19^3\left(\frac{1}{2}-\frac{1}{3}\right)=\frac{2}{3}19^3=\frac{13718}{3}$

29. Use Green's Theorem to evaluate the integral

$\int_C 13xydx+(x+y)dy = \iint_D (1-13x)dA = \int_{-13}^{13}\int_0^{169-x^2}(1-13x)dydx = \int_{-13}^{13}(1-13x)(169-x^2)dx$

for the path C: boundary of the region lying between the graphs of $y=0$ and $y=169-x^2$.

$\to \int_{-13}^{13} +13x^3-x^2-13^3x+169dx$ drop the odd terms

$=\int_{-13}^{13}-x^2+169\,dx=\left[\frac{-x^3}{3}+169x\right]_{-13}^{13}$

$=2\left(\frac{-13^3}{3}+13^3\right)=\frac{4}{3}13^3=\frac{8788}{3}$

30. Use Green's Theorem to evaluate the integral

$$\int_C (x^2 - y^2)\,dx + 10xy\,dy = \iint_{\bigcirc} 10y - (-2y)\,dA = \iint_{\bigcirc} 12y\,dA = \int_0^{2\pi}\int_0^3 12r^2 \sin\theta\,dr\,d\theta$$

for the path C: $x^2 + y^2 = 9$.

$$\Rightarrow \left[4r^3\right]_0^3 \left[-\cos\theta\right]_0^{2\pi} = 0$$

31. Use Green's Theorem to evaluate the integral

$$\int_C 8xy\,dx + 8(x+y)\,dy = 8\iint_{\odot} 1 - x\,dA = 8\left(\pi 4^2 - \pi 1^2\right) - \int_0^{2\pi}\int_1^4 r^2\cos\theta\,d\theta = 8\cdot 15\pi = 120\pi$$

$\to 0$ since $\int_0^{2\pi} \cos\theta\,d\theta = 0$

for C: boundary of the region lying between the graphs of $x^2 + y^2 = 1$ and $x^2 + y^2 = 16$.

32. Use Green's Theorem to calculate the work done by the force $\vec{\mathbf{F}}$ on a particle that is moving counterclockwise around the closed path C.

$\vec{\mathbf{F}}(x, y) = 4xy\hat{\mathbf{i}} + (x+y)\hat{\mathbf{j}}$

$C: x^2 + y^2 = 9$

$$\int_C F\cdot dr = \int 4xy\,dx + (x+y)\,dy = \iint_{\bigcirc} 1 - 4x\,dA$$

$$= \pi 3^2 - 4\int_0^{2\pi}\int_0^3 r^2\cos\theta\,d\theta = 9\pi$$

since $\int_0^{2\pi} \cos\theta\,d\theta = 0$

37. Find an equation of the tangent plane to the surface represented by the vector-valued function at the given point. Normal vector for the tangent plane is $r_u \times r_v$.

$\vec{\mathbf{r}}(u,v) = (10u+v)\hat{\mathbf{i}} + (u-v)\hat{\mathbf{j}} + v\hat{\mathbf{k}}, \quad (6,-6,6)$

$r_u = \langle 10, 1, 0\rangle, \quad r_v = \langle 1,-1,1\rangle, \quad r_u \times r_v = \begin{vmatrix} i & j & k \\ 10 & 1 & 0 \\ 1 & -1 & 1 \end{vmatrix} = \vec{i} - 10\vec{j} - 11\vec{k} \Rightarrow (x-6) - 10(y+6) - 11(z-6) = 0$

38. Find the area of the surface over the given region. Use a computer algebra system to verify your results.

The sphere,

$\vec{\mathbf{r}}(u,v) = 8\sin u \cos v\hat{\mathbf{i}} + 8\sin u \sin v\hat{\mathbf{j}} + 8\cos u\hat{\mathbf{k}}, \quad 0 \le u \le \pi, \quad 0 \le v \le 2\pi$

$r_u = \langle 8\cos u\cos v, 8\cos u \sin v, -8\sin u\rangle$

$r_v = \langle -8\sin u \sin v, 8\sin u\cos v, 0\rangle$

$r_u \times r_v = 64\langle \sin^2 u\cos v, \sin^2 u \sin v, \sin u\cos u\cos^2 v + \sin u\cos u \sin^2 v\rangle$

$r_u \times r_v = 64\sin u\langle \sin u\cos v, \sin u \sin v, \cos u\rangle$ so $\|r_u \times r_v\| = 64|\sin u| \Rightarrow \int_0^{\pi}\int_0^{2\pi} 64\sin u \, dv\, du = 2\cdot 64\cdot 2\pi = 256\pi$

39. Find the area of the surface over the given region. Use a computer algebra system to verify your results.

The part of the cone,

$\vec{\mathbf{r}}(u,v) = 3u\cos v\hat{\mathbf{i}} + 3u\sin v\hat{\mathbf{j}} + u\hat{\mathbf{k}}$,

where $0 \le u \le 8$ and $0 \le v \le 2\pi$.

$r_u = \langle 3\cos v, 3\sin v, 1\rangle$

$r_v = \langle -3u\sin v, 3u\cos v, 0\rangle$

$r_u \times r_v = \langle -3u\cos v, -3u\sin v, 9u\cos^2 v + 9u\sin^2 v\rangle$

$\|r_u \times r_v\| = 3\sqrt{u^2 + 9u^2} = 3\sqrt{10}|u|$

$\int_0^8\int_0^{2\pi} 3\sqrt{10}\, u\, dv\, du = 6\pi\sqrt{10}\left[\frac{u^2}{2}\right]_0^8 = 192\sqrt{10}\pi$

40. Evaluate $\iint_S (x-7y+z)dS$, where

$dS = \sqrt{1+g_x^2+g_y^2}\, dA = \sqrt{1+1^2+0}\, dA = \sqrt{2}\, dA$

S: $z = 14 - x, \quad 0 \le x \le 14, \quad 0 \le y \le 14$

$\sqrt{2}\int_0^{14}\int_0^{14}(x - 7y + 14 - x)\,dx\,dy = \sqrt{2}\int_0^{14}\int_0^{14}(14-7y)\,dx\,dy = 14\int_0^{14} 14 - 7y\,dy = 7\cdot 14\left[2y - \frac{y^2}{2}\right]_0^{14}\sqrt{2}$

$= -5\cdot 7\cdot 14^2\sqrt{2}$

$= -6860\sqrt{2}$

41. Evaluate $\iint_S f(x,y)dS$, where

$f(x,y) = y + 6$

$r_u = \langle 1, 0, 0\rangle$

$r_v = \langle 0, 1, \frac{1}{10}\rangle$

$r_u \times r_v = \langle 0, -\frac{1}{10}, 1\rangle$

$S: r(u,v) = u\mathbf{i} + v\mathbf{j} + \frac{v}{10}\mathbf{k}, \quad 0 \le u \le 1, \quad 0 \le v \le 10$

$\int_0^1\int_0^{10}(v+6)\|\langle 0, -\frac{1}{10}, 1\rangle\|\, dv\, du = \sqrt{1.01}\left[\frac{v^2}{2} + 6v\right]_0^{10} = \sqrt{1.01}\cdot 110$

42. Evaluate $\iint_S f(x,y,z)dS$, where

$f(x,y,z) = x^2 + y^2 + z^2$

$S: x^2 + y^2 = 4, \quad 0 \le z \le 4$

$r(\theta, z) = \langle 2\cos\theta, 2\sin\theta, z\rangle \quad r_\theta \times r_z = \langle 2\cos\theta, 2\sin\theta, 0\rangle$

$r_\theta = \langle -2\sin\theta, 2\cos\theta, 0\rangle \quad \|r_\theta \times r_z\| = 2$

$r_z = \langle 0, 0, 1\rangle$

$2\int_0^{2\pi}\int_0^4 4 + z^2\, dz\, d\theta = 4\pi\left[4z + \frac{z^3}{3}\right]_0^4 = \frac{16\cdot 28}{3}\pi = \frac{448}{3}\pi$