

Review from First Semester Calculus

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1-6 State the definition for each of the following

1. The function $f(x)$ is continuous at c $\lim_{x \rightarrow c} f(x) = f(c)$

3. The derivative of $f(x)$ with respect to x in terms of a limit:

$$\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

4. The definite integral of $f(x)$ on $[a, b]$ in terms of a limit:

The limit of the Riemann Sums as the norm of the partition goes to zero.

5. The indefinite integral of $f(x)$:

The most general antiderivative

6. $\ln(x)$ in terms of an integral:

$$\ln(x) = \int_1^x \frac{1}{t} dt \text{ for } x > 0$$

7. The average of $f(x)$ on the interval $[a, b]$:

$$\frac{1}{b-a} \int_a^b f(x) dx$$

8. State the Fundamental Theorem of Calculus:

For $f(x)$ continuous on $[a, b]$

$\int_a^b f(x) dx = F(b) - F(a)$ where $F(x)$ is an antiderivative of $f(x)$.

9. If $y = f(x)$ then

$$dy = f'(x) dx \quad \text{or} \quad \left(\frac{df(x)}{dx} \right) dx$$

10. If $y = f(x)$ then the equation of the tangent line at $x=a$ is--

$$y = f(a) + f'(a)(x-a)$$

$$11. \int x^n dx = \frac{x^{n+1}}{n+1} + c \text{ for } n \neq -1, \text{ see 14 for } n = -1$$

$$12. \int \sin(x) dx = -\cos(x) + c$$

$$13. \int \cos(x) dx = \sin(x) + c$$

$$14. \int \frac{1}{x} dx = \ln|x| + c$$

$$15. \int e^x dx = e^x + c$$

$$16. \frac{d}{dx} x^n = n x^{n-1}$$

$$17. \frac{d}{dx} \sin(x) = \cos(x)$$

$$18. \frac{d}{dx} \cos(x) = -\sin(x)$$

$$19. \frac{d}{dx} \tan(x) = \sec^2(x)$$

$$20. \frac{d}{dx} \sec(x) = \sec(x) \tan(x)$$

$$21. \frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$22. \frac{d}{dx} e^x = e^x$$

$$23. d(y^n) = n y^{n-1} dy$$

$$24. d(\sin(\theta)) = \cos(\theta) d\theta$$

$$25. d(\cos(z)) = -\sin(z) dz$$

$$26. d(\tan(\psi)) = \sec^2(\psi) d\psi$$

$$27. d(\sec(x)) = \sec(x) \tan(x) dx$$

$$28. d(\ln(u)) = \frac{du}{u}$$

$$29. d(e^x) = e^x dx$$

$$30. d(3\pi^2) = 0$$

31-35 Integrate the following:

$$31. \int \tan^3(x) \sec^2(x) dx = \int u^3 du = \frac{u^4}{4} + C = \frac{\tan^4 x}{4} + C$$

$u = \tan(x)$
 $du = \sec^2 x dx$

$$32. \int (\sin^3(x) + 1) \cos(x) dx = \int (u^3 + 1) du = \frac{u^4}{4} + u + C = \frac{\sin^4 x}{4} + \sin x + C$$

$u = \sin(x)$
 $du = \cos(x) dx$

$$33. \int \frac{t^2 + 2t + 1}{\sqrt{t+2}} dt = \int \frac{(u-2)^2 + 2(u-2) + 1}{\sqrt{u}} du = \int \frac{u^2 - 4u + 4 + 2u - 4 + 1}{\sqrt{u}} du$$

$u = t+2$
 $du = dt$
 $t = u-2$

$$= \int u^{3/2} - 2u^{1/2} + u^{-1/2} du = \frac{2u^{5/2}}{5} - \frac{4}{3}u^{3/2} + 2u^{1/2} + C$$
$$= \frac{2}{5}(t+2)^{5/2} - \frac{4}{3}(t+2)^{3/2} + 2(t+2)^{1/2} + C$$
$$= \frac{\sqrt{t+2}}{15} (6(t+2)^2 - 20(t+2) + 30) + C$$
$$= \frac{\sqrt{t+2}}{15} (6t^2 + 4t + 14) + C$$

34. $\int e^{\sin(x)} \cos(x) dx$
 $u = \sin(x)$
 $du = \cos(x) dx$
 $\int e^u du = e^u + C = e^{\sin(x)} + C$

$$35. \int_1^2 \frac{2x+1}{x^2+x} dx$$

$$u = x^2 + x$$

$$du = (2x+1) dx$$

$$\int_{x=1}^{x=2} \frac{du}{u} = \left[\ln|u| \right]_{x=1}^{x=2} = \ln 6 - \ln 2 = \ln \frac{6}{2} = \ln 3$$