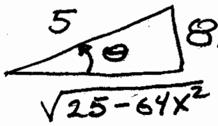
M251
P. Staley

Practice Exam 8.4-8.7

1. **Find the indefinite integral.**

$\int \sqrt{25-64x^2}\,dx$



$5\sin\theta = 8x$

$5\cos\theta\, d\theta = 8\,dx$

$5\cos\theta = \sqrt{25-64x^2}$

$\int 5\cos\theta \cdot \frac{5}{8}\cos\theta\, d\theta$

$= \frac{25}{8}\int \cos^2\theta\, d\theta$

$= \frac{25}{8}\int \frac{1+\cos 2\theta}{2}\, d\theta$

$\frac{25}{16}\left(\theta + \frac{1}{2}\sin 2\theta\right) + c$

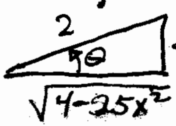
$\frac{25}{16}(\theta + \sin\theta\cos\theta)$

$= \frac{25}{16}\left(\arcsin\left(\frac{8x}{5}\right) + \frac{8x}{5}\cdot\frac{\sqrt{25-64x^2}}{5}\right)$

$\frac{25}{16}\arcsin\left(\frac{8x}{5}\right) + \frac{x\sqrt{25-64x^2}}{2} + c$

2. **Find the indefinite integral.**

$\int x\sqrt{4-25x^2}\,dx = \int \left(\frac{2}{5}\sin\theta\right)(2\cos\theta)\frac{2}{5}\cos\theta\, d\theta$



$2\sin\theta = 5x$

$2\cos\theta\, d\theta = 5\,dx$

$2\cos\theta = \sqrt{4-25x^2}$

$\frac{8}{25}\int \cos^2\theta\sin\theta\, d\theta$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$-\frac{8}{25}\int u^2\,du$

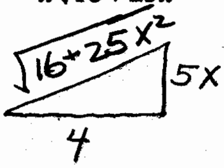
$\frac{-8}{25}\frac{u^3}{3} + c$

$= \frac{-8}{25}\cdot\frac{1}{3}\left(\frac{\sqrt{4-25x^2}}{2}\right)^3 + c$

$\frac{(25x^2-4)\sqrt{4-25x^2}}{75} + c$

3. **Find the indefinite integral.**

$\int \frac{1}{x\sqrt{16+25x^2}}\,dx = \int \frac{\frac{4}{5}\sec^2\theta\, d\theta}{\left(\frac{4}{5}\tan\theta\right)4\sec\theta}$



$4\tan\theta = 5x$

$4\sec^2\theta\, d\theta = 5\,dx$

$4\sec\theta = \sqrt{16+25x^2}$

$= \frac{1}{4}\int \csc\theta\, d\theta$

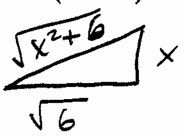
$= \frac{1}{4}\ln|\csc\theta - \cot\theta| + c$

$\frac{1}{4}\ln\left|\frac{\sqrt{16+25x^2}}{5x} - \frac{4}{5x}\right| + c$

$= \frac{1}{4}\ln\left|\frac{\sqrt{16+25x^2}-4}{5x}\right| + c$

4. Find the indefinite integral.

$$\int \frac{1}{\left(x^2+6\right)^{3/2}}\,dx = \int \frac{\sqrt{6}\sec^2\theta\,d\theta}{(\sqrt{6}\sec\theta)^3}$$



$$= \frac{1}{6}\int \cos\theta\,d\theta$$

$\sqrt{6}\tan\theta = x$

$$= \frac{1}{6}\sin\theta + C$$

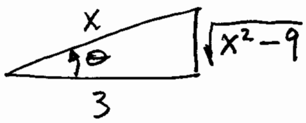
$\sqrt{6}\sec^2\theta\,d\theta = dx$

$\sqrt{x^2+6} = \sqrt{6}\sec\theta$

$$= \frac{x}{6\sqrt{x^2+6}} + C$$

5. Find the definite integral.

$$\int_3^4 \frac{\sqrt{x^2-9}}{x^2}\,dx = \int_{x=3}^{x=4} \frac{(3\tan\theta)\,3\sec\theta\tan\theta\,d\theta}{(3\sec\theta)^2}$$



$$= \int \frac{\tan^2\theta}{\sec\theta}\,d\theta$$

$$\int \sec\theta - \cos\theta\,d\theta$$

$3\sec\theta = x$

$$\left[\ln|\sec\theta+\tan\theta| - \sin\theta\right]$$

$3\sec\theta\tan\theta\,d\theta = dx$

$$= \left[\ln\left|\frac{x}{3}+\frac{\sqrt{x^2-9}}{3}\right| - \frac{\sqrt{x^2-9}}{x}\right]_3^4$$

$3\tan\theta = \sqrt{x^2-9}$

$$= \int \frac{\sec^2\theta - 1}{\sec\theta}\,d\theta$$

$$= \ln\left|\frac{4+\sqrt{7}}{3}\right| - \frac{\sqrt{7}}{4}$$

6. Find the indefinite integral.

$$\int \frac{x-28}{x^2-x-6}\,dx = \int \frac{-5}{x-3} + \frac{6}{x+2}\,dx$$

$$\frac{x-28}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$= -5\ln|x-3| + 6\ln|x+2| + C$$

or

$$\ln\left|\frac{(x+2)^6}{(x-3)^5}\right| + C$$

$x-28 = A(x+2) + B(x-3)$

when $x=3$ | when $x=-2$

$-25 = 5A$ | $-30 = -5B$

$-5 = A$ | $6 = B$

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7. **Find the indefinite integral.**

$$\int \frac{x^2}{x^2+2x-15}\,dx$$

$$\begin{array}{r} 1 \\ x^2+2x-15 \overline{) x^2} \\ -(x^2+2x-15) \\ \hline -2x+15 \end{array}$$

$$\frac{-2x+15}{(x+5)(x-3)} = \frac{A}{x+5} + \frac{B}{x-3}$$

$$-2x+15 = A(x-3)+B(x+5)$$

when $x=-5$: $25=-8A$, $-\frac{25}{8}=A$

when $x=3$: $9=8B$, $\frac{9}{8}=B$

$$\int 1+\frac{9}{8(x-3)}-\frac{25}{8(x+5)}\,dx$$

$$= x+\frac{9}{8}\ln|x-3|-\frac{25}{8}\ln|x+5|+C$$

8. **Find the indefinite integral.**

$$\int \frac{x^2+2x}{x^3-x^2+x-1}\,dx$$

$$\frac{x^2+2x}{(x-1)(x^2+1)} = \frac{A}{x-1}+\frac{Bx+C}{x^2+1}$$

$$x^2+2x = A(x^2+1)+(Bx+C)(x-1)$$

when $x=1$: $3=2A$, $\frac{3}{2}=A$

when $x=0$: $0=A-C$, $C=\frac{3}{2}$

when $x=-1$: $-1=2A+2B-2C$, $-\frac{1}{2}=B$

$$= \frac{1}{2}\int \frac{3}{x-1}-\frac{x-3}{x^2+1}\,dx$$

$$\frac{1}{2}\int \frac{3}{x-1}-\frac{x}{x^2+1}+\frac{3}{x^2+1}\,dx$$

$u=x^2+1$

$\frac{du}{2}=x\,dx$

$$\frac{1}{2}\left[3\ln|x-1|-\frac{1}{2}\ln|x^2+1|+3\arctan(x)\right]+C$$

9. **Find the indefinite integral.**

$$\int \frac{2x^3-5x^2+4x-4}{x^2-x}\,dx$$

$$\begin{array}{r} 2x-3 \\ x^2-x \overline{) 2x^3-5x^2+4x-4} \\ -(2x^3-2x^2) \\ \hline -3x^2+4x \\ -(-3x^2+3x) \\ \hline x-4 \end{array}$$

$$\frac{x-4}{x(x-1)} = \frac{A}{x}+\frac{B}{x-1}$$

$$x-4 = A(x-1)+Bx$$

when $x=0$: $-4=-A$, $4=A$

when $x=1$: $-3=B$

$$\int 2x-3+\frac{4}{x}-\frac{3}{x-1}\,dx$$

$$x^2-3x+4\ln|x|-3\ln|x-1|+C$$

10. Find the indefinite integral.

$$\int \frac{4x-2}{3(x-1)^2}\,dx$$

$$\frac{4x-2}{3(x-1)^2} = \frac{A}{3(x-1)} + \frac{B}{3(x-1)^2}$$

$4x-2 = A(x-1)+B$

when $x=1$: $2=B$

when $x=2$: $6=A+B$, $4=A$

$$\int \frac{4}{3(x-1)} + \frac{2}{3(x-1)^2}\,dx$$

$$= \frac{4}{3}\ln|x-1| + \frac{2}{3}\int u^{-2}\,du$$

$$= \frac{4}{3}\ln|x-1| - \frac{2}{3}u^{-1} + c$$

$$= \frac{4}{3}\ln|x-1| - \frac{2}{3(x-1)} + c$$

11. Use integration tables to evaluate the integral.

$$\int \frac{x}{(2+3x)^2}\,dx$$

use formula 4 with $a=2$ and $b=3$

$$\frac{1}{3^2}\left(\frac{2}{2+3x} + \ln|2+3x|\right) + c$$

$$= \frac{2}{9(2+3x)} + \frac{1}{9}\ln|2+3x| + c$$

12. Use integration tables to evaluate the integral.

$$\int \frac{x}{\sqrt{2+3x}}\,dx$$

use formula 21 with $a=2$ $b=3$

$$\frac{-2(2a-bu)}{3b^2}\sqrt{a+bu} + c$$

$$\frac{-2(4-3x)}{3\cdot 9}\sqrt{2+3x} + c$$

$$\frac{6x-8}{27}\sqrt{2+3x} + c$$

13. **Use integration tables to evaluate the integral.**

$$\int \frac{x}{1+\sin x^2}\,dx$$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{1+\sin(u)}$

using formula 56

$\frac{1}{2}(\tan u - \sec u) + c$

$\frac{1}{2}(\tan(x^2) - \sec(x^2)) + c$

14. **Use integration tables to evaluate the integral.**

$$\int \frac{x}{1+e^{x^2}}\,dx$$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{1+e^u}$

using formula 84

$\frac{1}{2}(u - \ln(1+e^u)) + c$

$\frac{x^2}{2} - \frac{1}{2}\ln(1+e^{x^2}) + c$

15. **Use integration tables to evaluate the integral.**

$$\int \frac{x}{x^2+4x+8}\,dx$$

using formula 15 with

$a=8,\ b=4,\ c=1$

$\frac{1}{2}\left(\ln|x^2+4x+8| - 4\int \frac{dx}{(x+2)^2+2^2}\right)$

$u = x+2$

$du = dx$

$\frac{1}{2}\left(\ln|x^2+4x+8| - 4\int \frac{du}{u^2+2^2}\right)$

using formula 23

$\frac{1}{2}\left(\ln(x^2+4x+8) - \frac{4}{2}\arctan\left(\frac{x+2}{2}\right) + c\right)$

$= \frac{1}{2}\ln(x^2+4x+8) - \arctan\left(\frac{x+2}{2}\right) + c$

16. Use integration tables to evaluate the integral.

$$\int \frac{3}{2x\sqrt{9x^2-1}}\,dx, \qquad x > 1/3$$

$u = 3x$

$du = 3dx$

$$\frac{3}{2}\int \frac{du}{u\sqrt{u^2-1}}$$

using formula 33

$\frac{3}{2}\ \text{arcsec}\,|u| + C$

$= \frac{3}{2}\ \text{arcsec}\,|3x| + C$

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17. Use integration tables to evaluate the integral.

$$\int \frac{1}{\sin \pi x \cos \pi x}\,dx$$

$u = \pi x$

$du = \pi dx$

$$\frac{1}{\pi}\int \frac{du}{\sin u \cos u}$$

using formula 58

$\frac{1}{\pi} \ln|\tan(\pi x)| + C$

18. Use integration tables to evaluate the integral.

$$\int \frac{1}{1+\tan \pi x}\,dx$$

$u = \pi x$

$du = \pi dx$

$$\frac{1}{\pi}\int \frac{du}{1+\tan u}$$

using formula 71

$\frac{1}{2\pi}\left(u + \ln|\cos u + \sin u|\right) + C$

$\frac{x}{2} + \frac{1}{2\pi} \ln|\cos \pi x + \sin \pi x| + C$

19. Use L'Hôpital's Rule to evaluate the limit.

$$\lim_{x\to 1}\frac{(\ln x)^2}{x-1} = \lim_{x\to 1}\frac{2(\ln x)\frac{1}{x}}{1} = 0$$

form $\frac{0}{0}$

20. Use L'Hôpital's Rule to evaluate the limit.

$$\lim_{x\to 0}\frac{\sin \pi x}{\sin 2\pi x} = \lim_{x\to 0}\frac{\pi\cos(\pi x)}{2\pi\cos(2\pi x)} = \frac{\pi(1)}{2\pi(1)} = \frac{1}{2}$$

form $\frac{0}{0}$

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21. Use L'Hôpital's Rule to evaluate the limit.

$$\lim_{x\to\infty}\frac{e^{2x}}{x^2} = \lim_{x\to\infty}\frac{2e^{2x}}{2x} = \lim_{x\to\infty}\frac{2e^{2x}}{1} = \infty$$

form $\frac{\infty}{\infty}$ form $\frac{\infty}{\infty}$

22. Use L'Hôpital's Rule to evaluate the limit.

$\lim_{x\to\infty} xe^{-x^2}$ $= \lim_{x\to\infty} \frac{x}{e^{x^2}} = \lim_{x\to\infty} \frac{1}{2xe^{x^2}} = 0$

form $\frac{\infty}{\infty}$

23. Use L'Hôpital's Rule to evaluate the limit.

$\lim_{x\to\infty}(\ln x)^{2/x}$ form $(\infty)^0$

$y = \lim_{x\to\infty} (\ln x)^{2/x}$

$\ln y = \lim_{x\to\infty} \ln\left[(\ln x)^{2/x}\right]$

$= \lim_{x\to\infty} \frac{2\ln(\ln x)}{x}$ form $\frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{\frac{2}{\ln x}\,\frac{1}{x}}{1} = 0$

$\ln y = 0$

$y = e^0 = \boxed{1}$

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24. Use L'Hôpital's Rule to evaluate the limit.

$\lim_{x\to 1^+}\left(\frac{2}{\ln x} - \frac{2}{x-1}\right)$

$\lim_{x\to 1^+}\left(\frac{2(x-1)}{(\ln x)(x-1)} - \frac{2\ln x}{(\ln x)(x-1)}\right)$

$\lim_{x\to 1^+} \frac{2(x-1-\ln x)}{(\ln x)(x-1)}$

form $\frac{0}{0}$

$2\lim_{x\to 1^+} \frac{1-\frac{1}{x}}{\frac{1}{x}(x-1)+\ln x}$

$= 2\lim_{x\to 1^+} \frac{x-1}{x-1+x\ln x}$

form $\frac{0}{0}$

$= 2\lim_{x\to 1^+} \frac{1}{1+\ln x+\frac{x}{x}}$

$= \frac{2}{2} = 1$

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