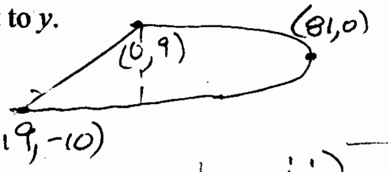
M251 Practice Exam Questions Larson 8th Ed Sections 7.1-7.3

1. Find the area of the region bounded by the equations by integrating (i) with respect to x and (ii) with respect to y.

$x = 81 - y^2$

$x = y - 9$



$81 - y^2 = y - 9 \Rightarrow y = 9$ or $9 + y = -1 \Rightarrow y = -10$

$y = 9 \Rightarrow x = 0 \qquad y = -10 \Rightarrow x = -19$

i) wrt x

$$\int_{-19}^{0} x + 9 + \sqrt{81 - x}\,dx + \int_{0}^{81} 2\sqrt{81 - x}\,dx$$

ii) wrt y

$$\int_{-10}^{9} 81 - y^2 - (y - 9)\,dy = \int_{-10}^{9} 90 - y - y^2\,dy$$

Area = 1143.17

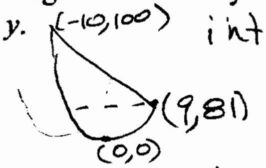
2. Find the area of the region bounded by equations by integrating (i) with respect to x and (ii) with respect to y.

$x = \pm\sqrt{y}$

$x = 90 - y$

$y = x^2$

$y = 90 - x$



intersection:

$x^2 = 90 - x \Rightarrow x^2 + x - 90 = 0$

$\Rightarrow (x + 10)(x - 9) = 0$

$x = -10$ or $x = 9$

$y = 100 \qquad y = 81$

i) wrt x

$$\int_{-10}^{9} 90 - x - x^2\,dx$$

ii) wrt y

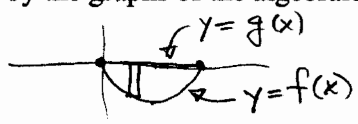
$$\int_{0}^{81} 2\sqrt{y}\,dy + \int_{81}^{100} 90 - y + \sqrt{y}\,dy$$

Area = 1143.17

3. Find the area of the region bounded by the graphs of the algebraic functions.

$f(x) = x^2 - 4x = x(x - 4)$

$g(x) = 0$



$$\int_{0}^{4} g(x) - f(x)\,dx = \int_{0}^{4} 4x - x^2\,dx = \left[2x^2 - \frac{x^3}{3}\right]_0^4 = 32 - \frac{64}{3} = \frac{32}{3} = 10.67$$

4. Find the area of the region bounded by the graphs of the algebraic functions.

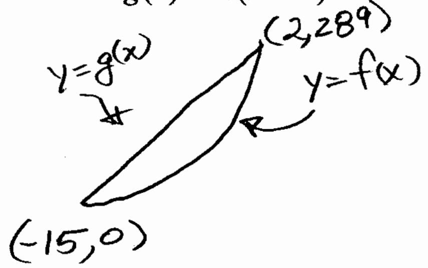
$f(x) = x^2 + 30x + 225 = (x + 15)^2$

$g(x) = 17(x + 15)$

intersection: $(x + 15)^2 = 17(x + 15)$

$\Rightarrow x = -15$ or $x + 15 = 17 \Rightarrow x = 2$

$y = 0 \qquad y = 17^2 = 289$



$$\int_{-15}^{2} 17(x + 15) - (x^2 + 30x + 225)\,dx$$

Area = 818.83

5. Find the area of the region bounded by the graphs of the algebraic functions.

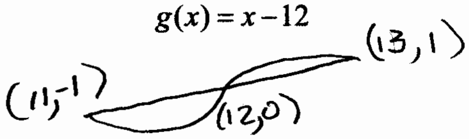
intersection: $\sqrt[3]{x-12} = x-12 \Rightarrow x-12 = (x-12)^3$

$\Rightarrow x = 12$ or $1 = (x-12)^2$

$\Rightarrow x = 13$ or $x = 11$

$f(x) = \sqrt[3]{x-12}$

$g(x) = x-12$



$\int_{11}^{12} x-12-\sqrt[3]{x-12}\,dx + \int_{12}^{13} \sqrt[3]{x-12} - x + 12\,dx = .25 + .25 = .50$

6. Find the area of the region bounded by the graphs of the algebraic functions.

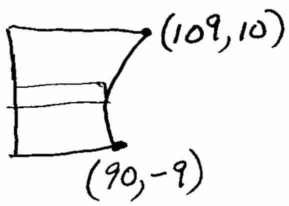
$x = f(y) = y^2+9, \quad g(y) = 0, \quad y = -9, \quad y = 10$

intersection: $y = 10$ and $x = y^2+9$

$\Rightarrow x = 109,\ y = 10$

$y = -9$ and $x = y^2+9$

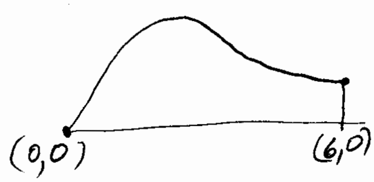
$\Rightarrow x = 90$



$\int_{-9}^{10} y^2+9\,dy = 747.33$

7. Find the area of the region bounded by the graphs of the equations.

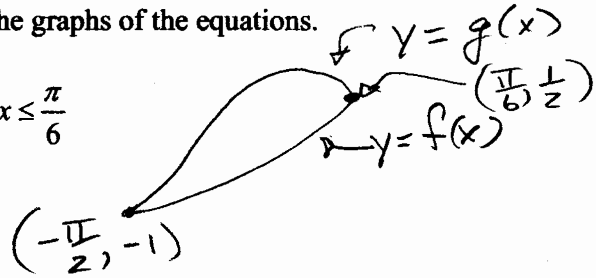
$f(x) = \frac{12x}{x^2+1}, \quad y = 0, \quad 0 \le x \le 6$



$\int_0^6 \frac{12x}{x^2+1}\,dx = 21.67$

8. Find the area of the region bounded by the graphs of the equations.

$f(x) = \sin(x), \quad g(x) = \cos(2x), \quad -\frac{\pi}{2} \le x \le \frac{\pi}{6}$



$\int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} (\cos(2x) - \sin(x))\,dx$

$\doteq 1.30$

9. Find the area of the region bounded by the graphs of the equations.

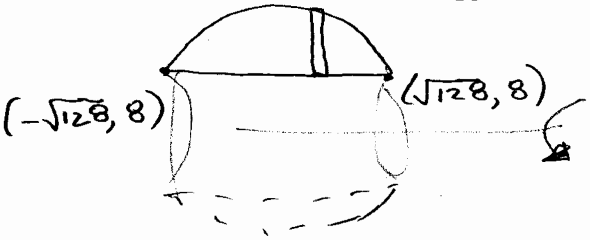
$f(x) = xe^{-x^2}, \quad y = 0, \quad 0 \le x \le 1$

$$\int_0^1 xe^{-x^2}\,dx = \frac{\left(1-\frac{1}{e}\right)}{2} = .316$$

10. Set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the x-axis.

$y = 8, y = 16 - \frac{x^2}{16}$

intersection: $8 = 16 - \frac{x^2}{16} \Rightarrow \frac{x^2}{16} = 8 \Rightarrow x^2 = 128 \Rightarrow x = \pm 8\sqrt{2}$

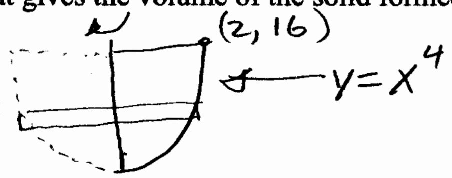


$$\pi \int_{-\sqrt{128}}^{\sqrt{128}} \left(\text{radius}_{out}^2 - \text{radius}_{in}^2\right) dx$$

$$= \pi \int_{-\sqrt{128}}^{\sqrt{128}} \left(\left(16 - \frac{x^2}{16}\right)^2 - 8^2\right) dx = 8492.42$$

11. Set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the y-axis.

$y = x^4, y = 16$ in the first quadrant

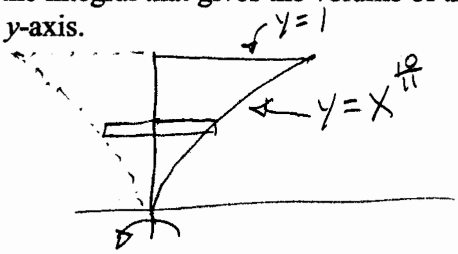


$$\pi \int \text{radius}^2\,dy$$

$$\pi \int_0^{16} \left(\sqrt[4]{y}\right)^2 dy = \frac{128\pi}{3} = 134.04$$

12. Set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the y-axis.

$y = x^{\frac{10}{11}}, y = 1, x = 0$



$\text{radius} = x = y^{\frac{11}{10}}$

$$\pi \int_0^1 \left(y^{\frac{11}{10}}\right)^2 dy = \pi \int_0^1 y^{\frac{11}{5}}\,dy = \left[\frac{5\pi}{16} y^{\frac{16}{5}}\right]_0^1 = \frac{5\pi}{16} = .98$$

13. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the given lines.

$y = x^2, y = 14x - x^2$

(i) x-axis; (ii) the line $y = 51$

intersection: $x^2 = 14x - x^2$

$\Rightarrow 2x^2 - 14x = 0$

$\Rightarrow 2x(x-7) = 0 \Rightarrow x = 0$ or $x = 7$

$\pi \int_{a}^{b} r_{outside}^2 - r_{inside}^2 \, dx$

i) $\pi \int_0^7 (14x - x^2)^2 - (x^2)^2 dx = 17600.25$

ii) $\pi \int_0^7 (51 - x^2)^2 - (51 - (14x - x^2))^2 dx$

$= 19037$

14. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the given lines.

$x = y^2, x = 14y - y^2$

(i) y-axis; (ii) the line $x = 51$

intersection: $y^2 = 14y - y^2$

$\Rightarrow 2y^2 - 14y = 0 \Rightarrow 2y(y-7) = 0$

$\Rightarrow y = 0 \quad y = 7$

$\pi \int_{a}^{b} r_{outside}^2 - r_{inside}^2 \, dy$

i) $\pi \int_0^7 (14y - y^2)^2 - (y^2)^2 dy = 17600.25$

ii) $\pi \int_0^7 (51 - y^2)^2 - (51 - (14y - y^2))^2 dy$

$= 19037$

15. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the given lines.

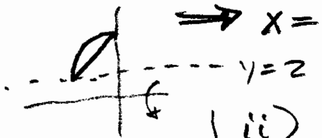
$y = 4 - 1x - x^2,\ y = x + 4$

(i) x-axis; (ii) the line $y = 2$

intersection: $4 - x - x^2 = x + 4$

$\Rightarrow x^2 + 2x = 0 \Rightarrow x(x+2) = 0$

$\Rightarrow x = 0$ or $x = -2$

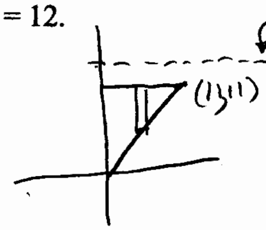


i) $\pi \int_{-2}^{0} (4 - x - x^2)^2 - (x+4)^2 dx = 28.48$

ii) $\pi \int_{-2}^{0} (4 - x - x^2 - 2)^2 - (x + 4 - 2)^2 dx = 11.73$

16. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line $y = 12$.

$y = x,\ y = 11,\ x = 0$



outside radius $= 12 - y$

$= 12 - x$

inside radius $= 12 - 11 = 1$

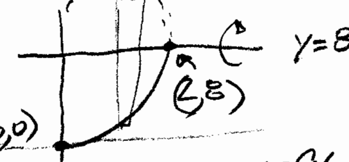
$\pi \int_0^{11} (12 - x)^2 - 1 \, dx = 1773.95$

17. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line $y = 8$.

$$y=\frac{1}{2}x^4,\ y=8,\ x=0$$

intersection: $\frac{1}{2}x^4 = 8 \Rightarrow x^4 = 16 \Rightarrow x = \pm 2$

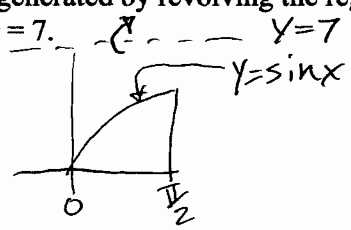
radius $= 8 - y = 8 - \frac{1}{2}x^4$



$$\pi \int_0^2 \left(8-\tfrac{1}{2}x^4\right)^2 dx = 285.95 = \frac{4096\pi}{45}$$

18. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line $y = 7$.

$$y=\sin x,\ y=0,\ 0 \le x \le \frac{\pi}{2}$$



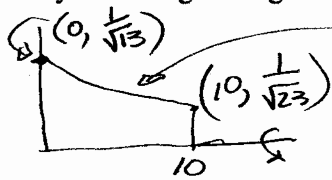
outside radius = 7

inside radius $= 7 - y$
$= 7 - \sin x$

$$\pi \int_0^{\frac{\pi}{2}} 7^2 - (7-\sin x)^2 dx = 41.51$$

19. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

$$y=\frac{1}{\sqrt{x+13}},\ y=0,\ x=0,\ x=10$$

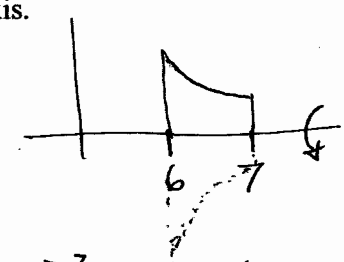


$Y = \frac{1}{\sqrt{x+13}}$ = radius

$$\pi \int_0^{10} \frac{1}{x+13} dx = \pi \left[\ln(x+13)\right]_0^{10} = \pi \ln \frac{23}{13} = 1.79$$

20. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

$$y=\frac{1}{x},\ y=0,\ x=6,\ x=7$$

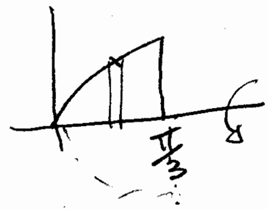


radius $= y = \frac{1}{x}$

$$\pi \int_6^7 \frac{1}{x^2} dx = \pi \left[\frac{-1}{x}\right]_6^7 = \pi\left(\frac{1}{6}-\frac{1}{7}\right) = .075$$

21. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis. Verify your results using the integration capabilities of a graphing utility.

$y = \sin(x)$, $y = 0$, $x = 0$, $x = \frac{\pi}{3}$



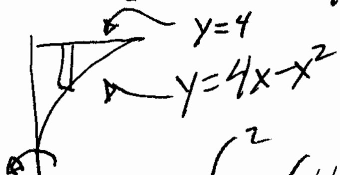
radius = $\sin(x)$

$$\pi \int_0^{\pi/3} \sin^2(x)\,dx = .96$$

22. Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the y-axis.

$y = 4x - x^2$, $x = 0$, $y = 4$

intersection: $4 = 4x - x^2 \Rightarrow x^2 - 4x + 4 = 0$
$\Rightarrow (x-2)^2 = 0 \Rightarrow x = 2$

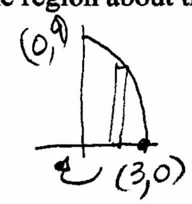


circumference $= 2\pi x$
height $= 4 - (4x - x^2)$
thickness $= dx$

$$2\pi \int_0^2 x\left(4 - (4x - x^2)\right)dx = 8.38$$

23. Use the shell method to set up and evaluate an integral that gives the volume of the solid generated by revolving the plane region about the y-axis.

$y = 9 - x^2$, $y = 0$, $x = 0$



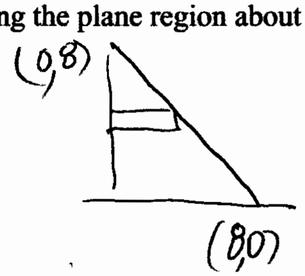
circumference $= 2\pi x$
height $= 9 - x^2$
thickness $= dx$

$$2\pi \int_0^3 x(9 - x^2)\,dx = 2\pi\left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_0^3$$

$$= \frac{81\pi}{2} = 127.23$$

24. Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the x-axis.

$y = 8 - x$, $y = 0$, $x = 0$



circumference $= 2\pi y$
height $= x = 8 - y$
thickness $= dy$

$$2\pi \int_0^8 y(8 - y)\,dy$$

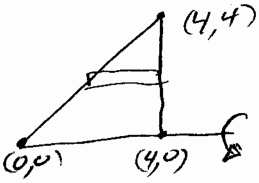
$$= 2\pi\left[4y^2 - \frac{y^3}{3}\right]_0^8$$

$$= 2\pi\, 8^3\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{8^3\pi}{3}$$

$$= 536.17$$

25. Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the x-axis.

$y = x,\ y = 0,\ x = 4$



circumference = $2\pi y$
height = $4 - x = 4 - y$
thickness = dy

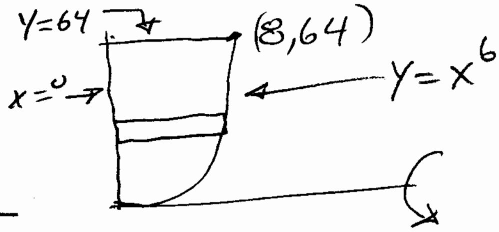
$$2\pi\int_0^4 y(4-y)\,dy = 2\pi\left[2y^2 - \frac{y^3}{3}\right]_0^4$$

$$2\pi 4^3\left(\frac{1}{2}-\frac{1}{3}\right)$$

$$= \frac{64\pi}{3} = 67.02$$

26. Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the x-axis.

$y = x^6,\ x = 0,\ y = 64$



circumference = $2\pi y$
height = $x = \sqrt[6]{y}$
thickness = dy

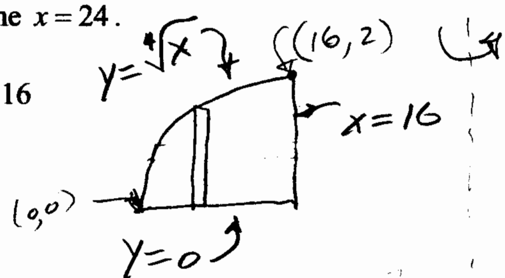
$$2\pi\int_0^{64} y\sqrt[6]{y}\,dy = 2\pi\left[\frac{6}{13}y^{\frac{13}{6}}\right]_0^{64}$$

$$\frac{12}{13}\pi 2^{13}$$

$$= 23756.24$$

27. Use the shell method to find the volume of the solid generated by revolving the plane region about the line $x = 24$.

$y = \sqrt[4]{x},\ y = 0,\ x = 16$



circumference = $2\pi(24 - x)$
height = $y = \sqrt[4]{x}$
thickness = dx

$$2\pi\int_0^{16}(24-x)\sqrt[4]{x}\,dx =$$

$$= 2\pi\left[24\cdot\frac{4}{5}x^{\frac{5}{4}} - \frac{4}{9}x^{\frac{9}{4}}\right]_0^{16} = 2430.62$$

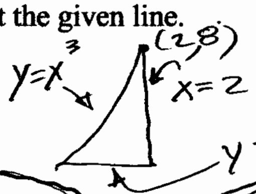
REVISED
5:25 am, 2/14/07

DISK METHOD

28. Use the disk *or* shell method to find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the given line.

$y = x^3$, $y = 0$, $x = 2$

region:



(i) the x-axis; (ii) the y-axis; (iii) the line $x = 4$

i) radius $= x^3$

$$\pi\int_0^2 (x^3)^2\,dx = \frac{\pi}{7}2^7 = 57.45$$

ii) $r_{out} = 2$

$r_{in} = x = \sqrt[3]{y}$

$$\pi\int_0^8 4 - (\sqrt[3]{y})^2\,dy = \pi\left[4y - \frac{3y^{5/3}}{5}\right]_0^8 = 40.21$$

iii) $x=2$, $x=4$

$r_{out} = 4 - x = 4 - \sqrt[3]{y}$

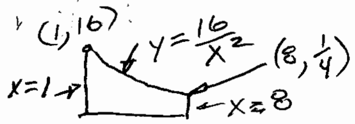
$r_{in} = 2$

$$\pi\int_0^8 (4-\sqrt[3]{y})^2 - 4\,dy$$

$= 60.32$

29. Use the disk *or* shell method to find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the given line.

$y = \dfrac{16}{x^2}$, $y = 0$, $x = 1$, $x = 8$



(i) the x-axis; (ii) the y-axis; (iii) the line $y = 16$

i) radius $= y = \frac{16}{x^2}$

$$\pi\int_1^8 \left(\frac{16}{x^2}\right)^2 dx = 267.56$$

ii) $r_{out} = 8$ or $x = \frac{4}{\sqrt{y}}$

$r_{in} = 1$

$$\pi\int_0^{1/4} 8^2 - 1^2\,dy + \pi\int_{1/4}^{16}\left(\frac{4}{\sqrt{y}}\right)^2 - 1^2\,dy$$

$= 209.05$

iii) $y=16$

$r_{out} = 16$

$r_{in} = 16 - y = 16 - \frac{16}{x^2}$

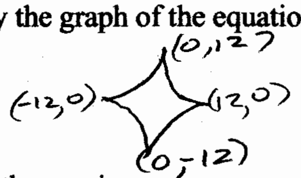
$$\pi\int_1^8 16^2 - \left(16 - \frac{16}{x^2}\right)^2 dx$$

$$256\pi\int_1^8 1 - \left(1 - \frac{1}{x^2}\right)^2 dx$$

$= 1139.87$

30. Use the disk *or* shell method to find the volume of the solid generated by revolving the region bounded by the graph of the equation about the given line.

$x^{\frac{2}{3}} + y^{\frac{2}{3}} = 12^{\frac{2}{3}}$



(i) the x-axis; (ii) the y-axis

i) radius $= y = (12^{2/3} - x^{2/3})^{3/2}$

$$\pi\int_{-12}^{12} (12^{2/3} - x^{2/3})^3\,dx$$

$= 1654.45$

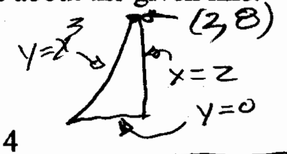
ii) radius $= x = (12^{2/3} - y^{2/3})^{3/2}$

$$\pi\int_{-12}^{12} (12^{2/3} - y^{2/3})^3\,dy = 1654.45$$

SHELL METHOD

28. Use the disk *or* shell method to find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the given line.

$y = x^3$, $y = 0$, $x = 2$ region:



(i) the x-axis; (ii) the y-axis; (iii) the line $x = 4$

i) $2\pi\int_0^8 y(2-\sqrt[3]{y})\,dy = 57.45$

circumference $= 2\pi y$

height $= 2 - x = 2 - \sqrt[3]{y}$

thickness $= dy$

ii) $2\pi\int_0^2 x(x^3)\,dx = \frac{64\pi}{5} = 40.21$

circ. $= 2\pi x$

height $= y = x^3$

thickness $= dx$

iii) $x=4$

radius

circ. $= 2\pi(4-x)$

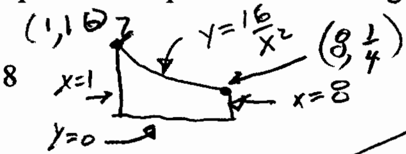
height $= y = x^3$

thickness $= dx$

$2\pi\int_0^2 (4-x)x^3\,dx = \frac{3.32\pi}{5} = 60.32$

29. Use the disk *or* shell method to find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the given line.

$y = \frac{16}{x^2}$, $y = 0$, $x = 1$, $x = 8$



(i) the x-axis; (ii) the y-axis; (iii) the line $y = 16$

i) $2\pi\int_0^{\frac{1}{4}} 7y\,dy + \int_{\frac{1}{4}}^{16} 2\pi y\left(\frac{4}{\sqrt{y}}-1\right)dy$

circ. $= 2\pi y$

height $= 8-1$ or $x-1 = \frac{4}{\sqrt{y}} - 1$

thickness $= dy$

267.56

ii) $2\pi\int_1^8 x\,\frac{16}{x^2}\,dx = 32\pi\ln 8 = 209.05$

circum $= 2\pi x$

height $= y = \frac{16}{x^2}$

thickness $= dy$

iii) $y=16$

circum $= 2\pi(16-y)$

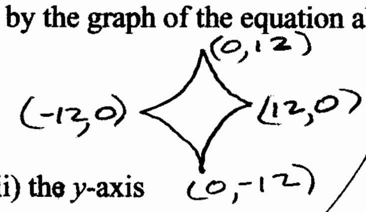
height $= 7$ or $x-1 = \frac{4}{\sqrt{y}} - 1$

thickness $= dy$

$2\pi\int_0^{\frac{1}{4}} (16-y)7\,dy + 2\pi\int_{\frac{1}{4}}^{16} (16-y)\left(\frac{4}{\sqrt{y}}-1\right)dy = 1139.87$

30. Use the disk *or* shell method to find the volume of the solid generated by revolving the region bounded by the graph of the equation about the given line.

$x^{\frac{2}{3}} + y^{\frac{2}{3}} = 12^{\frac{2}{3}}$



(i) the x-axis; (ii) the y-axis

i)

circumference $= 2\pi y$

height $= 2x = 2\sqrt{(12^{2/3} - y^{2/3})^3}$

thickness $= dy$

$4\pi\int_0^{12} y\sqrt{(12^{2/3} - y^{2/3})^3}\,dy = 1654.45$

ii)

circumference $= 2\pi x$

height: $2y = 2(12^{2/3} - x^{2/3})^{3/2}$

thickness $= dx$

$4\pi\int_0^{12} x(12^{2/3} - x^{2/3})^{3/2}\,dx = 1654.45$