

For every problem that states "determine the convergence or divergence of the series" also justify your answer by identifying the theorem or test and showing how the condition or conditions were satisfied.

1. Write the first five terms of the sequence.

$$a_n = \left(-\frac{4}{5}\right)^n$$

$$-\frac{4}{5}, \frac{16}{25}, -\frac{64}{125}, \frac{256}{625}, -\frac{1024}{3125}$$

2. Write the first five terms of the sequence.

$$a_n = (-1)^{n+4} \left(\frac{17}{n}\right)$$

$$-17, \frac{17}{2}, -\frac{17}{3}, \frac{17}{4}, -\frac{17}{5}$$

3. Write the first five terms of the sequence.

$$a_n = 5 - \frac{3}{n} - \frac{7}{n^2}$$

$$-5, \frac{7}{4}, \frac{29}{9}, \frac{65}{16}, \frac{103}{25}$$

4. Determine the convergence or divergence of the sequence with the given n th term. If the sequence converges, find its limit.

$$a_n = \frac{\ln(n^{10})}{6n}$$

$\frac{\infty}{\infty}$ form

\rightarrow L'Hopital's rule

$$= \lim_{n \rightarrow \infty} \frac{10 \frac{1}{n}}{6} = 0 \text{ converges}$$

5. Determine the convergence or divergence of the sequence with the given n th term. If the sequence converges, find its limit.

$$a_n = \frac{\ln(\sqrt[3]{n})}{8n} = \frac{\ln n}{9.8n} \quad \frac{\infty}{\infty} \text{ form, by L'Hopital's Rule:}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{1/n}{72} = 0$$

6. Determine the convergence or divergence of the sequence with the given n th term. If the sequence converges, find its limit.

$$a_n = \frac{2^n}{5^n} = \left(\frac{2}{5}\right)^n \rightarrow 0$$

7. Write the first five terms of the sequence of partial sums.

$$5 + \frac{5}{4} + \frac{5}{9} + \frac{5}{16} + \frac{5}{25} + \dots$$

$$S_1 = 5$$

$$S_2 = 5 + \frac{5}{4} = 6.25$$

$$S_3 = S_2 + \frac{5}{9} = \frac{245}{36}$$

$$S_4 = \frac{1025}{144}$$

$$S_5 = \frac{5269}{720}$$

8. Write the first five terms of the sequence of partial sums.

$$-5 + \frac{25}{6} - \frac{125}{36} + \frac{625}{216} - \frac{3125}{1296} + \dots$$

$$S_1 = -5$$

$$S_2 = -5/6$$

$$S_3 = -155/36$$

$$S_4 = -305/216$$

$$S_5 = -4955/1296$$

9. Write the first five terms of the sequence of partial sums.

$$\sum_{n=1}^{\infty} \frac{5}{(4)^{n-1}}$$

$$S_1 = 5$$

$$S_2 = S_1 + \frac{5}{4} = 6.25$$

$$S_3 = S_2 + \frac{5}{16} = \frac{105}{16}$$

$$S_4 = S_3 + \frac{5}{64} = \frac{425}{64}$$

$$S_5 = S_4 + \frac{5}{256} = \frac{1705}{256}$$

10. Find the sum of the convergent series.

$$\sum_{n=1}^{\infty} \frac{6}{(n+4)(n+6)}$$

$$\frac{6}{(n+4)(n+6)} = \frac{A}{n+4} + \frac{B}{n+6}$$

$$6 = (n+6)A + (n+4)B$$

$$n = -4 \Rightarrow 6 = 2A \Rightarrow 3 = A$$

$$n = -6 \Rightarrow 6 = -2B \Rightarrow -3 = B$$

$3 \sum_{n=1}^{\infty} \frac{1}{n+4} - \frac{1}{n+6}$ is a telescoping series

$$3 \left(\frac{1}{5} - \frac{1}{7} + \frac{1}{6} - \frac{1}{8} + \frac{1}{7} - \frac{1}{9} + \frac{1}{8} - \frac{1}{10} + \dots \right)$$

$$S_n = 3 \left(\frac{1}{5} + \frac{1}{6} - \frac{1}{n+5} - \frac{1}{n+6} \right) \rightarrow \frac{33}{30} = \frac{11}{10}$$

11. Find the sum of the convergent series.

$$\sum_{n=1}^{\infty} (-1)^n \frac{4}{(n+9)(n+11)}$$

$$\frac{4}{(n+9)(n+11)} = \frac{A}{n+9} + \frac{B}{n+11}$$

$$4 = (n+11)A + (n+9)B$$

$$n = -9 \Rightarrow 4 = 2A \Rightarrow 2 = A$$

$$n = -11 \Rightarrow 4 = -2B \Rightarrow -2 = B$$

$2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n+9} - \frac{(-1)^n}{n+11}$ is a telescoping series

$$2 \left(-\frac{1}{10} + \frac{1}{12} + \frac{1}{11} - \frac{1}{13} - \frac{1}{12} + \frac{1}{14} + \frac{1}{13} - \frac{1}{15} - \frac{1}{14} + \frac{1}{16} + \dots \right)$$

$$S_n = 2 \left(\frac{1}{10} + \frac{1}{11} - \frac{1}{n+10} - \frac{1}{n+11} \right) \rightarrow 2 \left(\frac{1}{10} + \frac{1}{11} \right) = \frac{2}{55}$$

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12. Find the sum of the convergent series.

$\sum_{n=0}^{\infty} 9 \left(\frac{10}{11} \right)^n$ geometric series with $r = \frac{10}{11} < 1$
converges to $\frac{\text{first}}{1 - \text{ratio}} = \frac{9}{1 - \frac{10}{11}} = 99$

13. Find the sum of the convergent series.

$\sum_{n=0}^{\infty} 2 \left(-\frac{9}{10} \right)^n$ geometric series with $-1 < r = -\frac{9}{10} < 1$
converges to $\frac{\text{first}}{1 - \text{ratio}} = \frac{2}{1 - -\frac{9}{10}} = \frac{20}{19}$

14. Determine the convergence or divergence of the series.

$\sum_{n=1}^{\infty} \frac{4^{-n}}{n^4} = \sum_{n=1}^{\infty} \frac{n^4}{4^n}$ using the ratio test we have
 $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)^4}{4^{n+1}}}{\frac{n^4}{4^n}} = \lim_{n \rightarrow \infty} \left(\frac{n+1}{n} \right)^4 \frac{4^n}{4^{n+1}} = \frac{1}{4}$

so the series converges

15. Determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \frac{2}{2^n}$$

This is a geometric series with $r = \frac{1}{2} \Rightarrow |r| < 1 \Rightarrow$ series converges

16. Find all values of x for which the series converges. For these values of x , write the sum of the series as a function of x .

$$\sum_{n=0}^{\infty} \frac{x^n}{9^n}$$

This is a geometric series with $r = \frac{x}{9}$ which converges for $-1 < \frac{x}{9} < 1$ i.e. $-9 < x < 9$. For those values of x the series converges to $\frac{\text{first}}{1-\text{ratio}} = \frac{1}{1-\frac{x}{9}} = \frac{9}{9-x}$

17. Find all values of x for which the series converges. For these values of x , write the sum of the series as a function of x .

$$\sum_{n=0}^{\infty} 10 \left(\frac{x-4}{10} \right)^n$$

This is a geometric series with $r = \frac{x-4}{10}$

which converges for $-1 < \frac{x-4}{10} < 1$
 $\Rightarrow -10 < x-4 < 10 \Rightarrow -6 < x < 14$. For those values the series converges to $\frac{\text{first}}{1-\text{ratio}} = \frac{10}{1-\frac{x-4}{10}} = \frac{100}{14-x}$

18. Use the Integral Test to determine the convergence or divergence of the series. Show your work.

$$\sum_{n=1}^{\infty} \frac{7}{10n+2}$$

Let $f(x) = \frac{7}{10x+2}$ then $f'(x) = \frac{-70}{(10x+2)^2} < 0$

so $f(x)$ is differentiable, continuous, and decreasing, \Rightarrow series and $\int_1^{\infty} f(x) dx$ converge or diverge together.

$$\int_1^{\infty} \frac{7}{10x+2} dx = \lim_{b \rightarrow \infty} \left[\frac{7}{10} \ln(10x+2) \right]_1^b = \frac{7}{10} \left(\lim_{b \rightarrow \infty} (10b+2) - \ln 12 \right) = \infty$$

so the series diverges

19. Use the Integral Test to determine the convergence or divergence of the series. Show your work.

Let $f(x) = x e^{-\frac{x}{2}}$ then $f'(x) = e^{-\frac{x}{2}} - \frac{x}{2} e^{-\frac{x}{2}} = (1 - \frac{x}{2}) e^{-\frac{x}{2}} < 0$

$$\sum_{n=1}^{\infty} n e^{-\frac{n}{2}}$$

so $f(x)$ is continuous and decreasing.

Thus by the integral test the series converges or diverges with $\int_1^{\infty} x e^{-\frac{x}{2}} dx$ integrating by parts $u=x$ $dv=e^{-\frac{x}{2}} dx$
 $bdu=dx$ $v=-2e^{-\frac{x}{2}}$

20. Use the Integral Test to determine the convergence or divergence of the series. Show your work.

let $f(x) = \frac{\ln x}{x^7}$, $f'(x) = \frac{\frac{1}{x} x^7 - 7x^6 \ln x}{x^{14}} = \frac{1-7\ln x}{x^8} < 0$

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^7}$$

so $f(x)$ is continuous and decreasing.

$\int \frac{\ln x}{x^7} dx$ integrating by parts $u=\ln x$ $dv=\frac{dx}{x^7}$
 $du=\frac{dx}{x}$ $v=-\frac{1}{6x^6}$

$= -\frac{\ln x}{6x^6} - \int \frac{dx}{-6x^7} = -\frac{\ln x}{6x^6} - \frac{1}{36x^6}$ For the limit \rightarrow use L'Hopital's rule

21. Use the Integral Test to determine the convergence or divergence of the series. Show your work.

$f(x) = \frac{4}{x\sqrt{\ln x}}$ is continuous and decreasing.

$$\sum_{n=2}^{\infty} \frac{4}{n\sqrt{\ln n}}$$

$\int \frac{4}{x\sqrt{\ln x}} dx$ substitute $u=\ln x$, $du=\frac{dx}{x}$

$\int \frac{4}{\sqrt{u}} du = 8\sqrt{u} = 8\sqrt{\ln x}$; $\int_2^{\infty} \frac{4}{x\sqrt{\ln x}} dx = \lim_{b \rightarrow \infty} [8\sqrt{\ln x}]_2^b \rightarrow \infty$

so the series diverges

22. Use the p-series theorem to determine the convergence or divergence of the series.

$\sum_{n=1}^{\infty} \frac{8}{n^{\frac{10}{7}}} = 8 \sum_{n=1}^{\infty} \frac{1}{n^{\frac{10}{7}}}$ which is a p-series with $p = \frac{10}{7} > 1$

so the series converges.

23. Use the p-series theorem to determine the convergence or divergence of the series.

$1 + \frac{1}{\sqrt[3]{2^2}} + \frac{1}{\sqrt[3]{3^2}} + \frac{1}{\sqrt[3]{4^2}} + \frac{1}{\sqrt[3]{5^2}} + \dots = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{2}{3}}}$ which is a p-series

with $p = \frac{2}{3} < 1$ so the series diverges

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24. Use the p-series theorem to determine the convergence or divergence of the series.

$\sum_{n=1}^{\infty} \frac{1}{n^{0.78}}$ This is a p series with $p = 0.78 < 1$ so
the series diverges

25. Determine the convergence or divergence of the series.

$\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$ clearly $1 < 3n^2 \Rightarrow 0 < 3n^2-1 \Rightarrow n^2 < 4n^2-1$
 $\Rightarrow \frac{1}{4n^2-1} < \frac{1}{n^2}$. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent

p-series ($p=2 > 1$) so by D.C.T. $\sum \frac{1}{4n^2-1}$ converges

soln 24 Using the LCT with $\sum \frac{1}{n^2}$ $\lim_{n \rightarrow \infty} \frac{\frac{1}{n^2}}{\frac{1}{4n^2-1}} = \lim_{n \rightarrow \infty} \frac{4n^2-1}{n^2} = \lim_{n \rightarrow \infty} 4 - \frac{1}{n^2} = 4$.

26. Determine the convergence or divergence of the series.

$\sum_{n=1}^{\infty} \frac{7}{n \cdot \sqrt[3]{n}} = 7 \sum_{n=1}^{\infty} \frac{1}{n^{1.125}}$ which is

a p series with $p = \frac{9}{8} > 1$ so

the series converges

$\sum \frac{1}{n^2}$ converges
(p series w $p=2 > 1$)
so $\sum \frac{1}{4n^2-1}$ converges

27. Determine the convergence or divergence of the series.

$3 \cdot \sum_{n=1}^{\infty} \frac{1}{n^{0.95}}$ this is a p-series with $p = 0.95 < 1$
so the series diverges.

28. Determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \left(\frac{5}{3}\right)^n \quad \text{This is a geometric series with } r = \frac{5}{3} > 1 \text{ so the series diverges.}$$

29. Use the Ratio Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} n \left(\frac{3}{4}\right)^n \quad \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{(n+1) \left(\frac{3}{4}\right)^{n+1}}{n \left(\frac{3}{4}\right)^n} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) \frac{3}{4} = \frac{3}{4}$$

$$\frac{3}{4} < 1 \Rightarrow \text{series converges}$$

30. Use the Ratio Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{n^9}{4^{-n}} \quad \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{(n+1)^9 4^{n+1}}{n^9 4^n} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^9 4 = 4$$

$$4 > 1 \text{ so the series diverges}$$

31. Use the Ratio Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \left(\frac{10}{8}\right)^n}{n^2} \quad \lim_{n \rightarrow \infty} \left| \frac{(-1)^n \left(\frac{10}{8}\right)^{n+1}}{(n+1)^2} \div \frac{(-1)^{n-1} \left(\frac{10}{8}\right)^n}{n^2} \right| = \lim_{n \rightarrow \infty} \left(\frac{n}{n+1}\right)^2 \frac{10}{8} = \frac{10}{8}$$

$$\frac{10}{8} > 1 \Rightarrow \text{series diverges}$$

32. Use the Root Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \left(\frac{8n}{8n+1}\right)^n \quad \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \left(\frac{8n}{8n+1}\right) = \lim_{n \rightarrow \infty} \left(\frac{1}{1 + \frac{1}{8n}}\right) = 1$$

so the Root test is inconclusive.

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33. Use the Root Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \left(\frac{4n+1}{8n-1} \right)^n \quad \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \frac{4n+1}{8n-1} = \lim_{n \rightarrow \infty} \frac{4 + \frac{1}{n}}{8 - \frac{1}{n}} = \frac{4}{8} = \frac{1}{2} < 1$$

⇒ series converges

34. Use the Root Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \left(\frac{7n^2+1}{4n^2-1} \right)^n \quad \lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \frac{7n^2+1}{4n^2-1} = \lim_{n \rightarrow \infty} \frac{7 + \frac{1}{n^2}}{4 - \frac{1}{n^2}} = \frac{7}{4} > 1$$

⇒ series diverges

35. Determine the convergence or divergence of the series using any appropriate test from this chapter. Identify the test used.

From the alternating series test

$$\sum_{n=1}^{\infty} \frac{(-1)^n 8}{3n} \quad \text{If } f(x) = \frac{8}{3x} \text{ then } f'(x) = -\frac{8}{3x^2} < 0$$

$f' < 0 \Rightarrow$ terms are decreasing; $\lim_{n \rightarrow \infty} \frac{8}{3n} = 0$

∴ this series converges by the A.S.T.

36. Determine the convergence or divergence of the series using any appropriate test from this chapter. Identify the test used.

$$\sum_{n=1}^{\infty} \frac{6}{n^2} = 6 \sum_{n=1}^{\infty} \frac{1}{n^2} \text{ which is a } p\text{-series with } p=2 > 1 \text{ hence it converges by } p\text{-series test}$$

or using the integral test $f(x) = \frac{6}{x^2}$ is

diff. and decreasing on $x \geq 1$. Furthermore

$$\int_1^{\infty} \frac{6}{x^2} dx = \lim_{b \rightarrow \infty} \left[-\frac{6}{x} \right]_1^b = 6 \text{ so the series converges.}$$