

Name: _____

Key

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1. Find the indefinite integral.

$$\int 8(x-5)^7 dx$$

$$u = x - 5$$

$$du = dx$$

$$\int 8u^7 du = \frac{8u^8}{8} + c = u^8 + c = (x-5)^8 + c$$

2. Find the indefinite integral.

$$\int \frac{2}{(r-1)^4} dr$$

$$u = r - 1$$

$$du = dr$$

$$\int \frac{2}{u^4} du = \int 2u^{-4} du = \frac{2u^{-3}}{-3} + c = -\frac{2}{3}u^{-3} + c = -\frac{2}{3(r-1)^3} + c$$

3. Find the indefinite integral.

$$\int \left[r - \frac{2}{(r-5)^8} \right] dr$$

$$= \int r dr - 2 \int \frac{dr}{(r-5)^8}$$

$$u = r - 5$$

$$du = dr$$

$$\frac{r^2}{2} - 2 \int u^{-8} du = \frac{r^2}{2} - \frac{2u^{-7}}{-7} + c = \frac{r^2}{2} + \frac{2}{7(r-5)^7} + c$$

4. Find the indefinite integral.

$$\int \frac{z^2}{z+7} dz$$

$$u = z + 7$$

$$du = dz$$

$$u - 7 = z$$

$$\int \frac{(u-7)^2}{u} du = \int \frac{u^2 - 14u + 49}{u} du$$

$$= \int \left(u - 14 + \frac{49}{u} \right) du = \frac{u^2}{2} - 14u + 49 \ln|u|$$

$$= \frac{(z+7)^2}{2} - 14z + 49 \ln|z+7| + c$$

$$= \frac{(z-7)^2}{2} + 49 \ln|z+7| + c$$

5. Find the indefinite integral.

$$\int \frac{6y}{y+3} dy$$

$$u = y + 3$$

$$du = dy$$

$$u - 3 = y$$

$$\int \frac{6(u-3)}{u} du = \int \left(6 - \frac{18}{u} \right) du$$

$$= 6u - 18 \ln|u| + c$$

$$= 6y - 18 \ln|y+3| + c$$

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6. Find the indefinite integral.

$$\int \frac{\cot(12/w^2)}{w^3} dw$$

$$u = \frac{12}{w^2}$$

$$du = -\frac{24}{w^3} dw$$

$$\frac{du}{-24} = \frac{dw}{w^3}$$

$$-\frac{1}{24} \int \cot(u) du$$

$$-\frac{\ln|\sin u|}{24} + C = -\frac{1}{24} \ln|\sin(\frac{12}{w^2})| + C$$

7. Find the definite integral.

$$\int_0^{15} \frac{4x}{\sqrt{x^2+64}} dx$$

$$u = x^2 + 64$$

$$du = 2x dx$$

$$\int_{x=0}^{x=15} \frac{2du}{\sqrt{u}} = \left[2u^{\frac{1}{2}} \right]_{x=0}^{x=15}$$

$$x=15 \quad u=289$$

$$x=0 \quad u=64$$

$$= 4 \left[\sqrt{289} + \sqrt{64} \right]$$

$$= 4(17+8) = 100$$

8. Find the definite integral.

$$\int_0^2 x^3 e^{-x^4} dx$$

$$u = -x^4$$

$$du = -4x^3 dx$$

$$-\frac{1}{4} \int_{x=0}^{x=2} e^u du = \left[-\frac{1}{4} e^u \right]_{x=0}^{x=2} = -\frac{1}{4} (e^{-16} - e^0) = \frac{1}{4} - \frac{1}{4} e^{-16}$$

9. Find the indefinite integral.

$$\int x^3 e^{6x^2} dx$$

$$w = 6x^2$$

$$dw = 12x dx$$

$$\frac{dw}{12} = x dx$$

$$\frac{w}{6} = x^2$$

$$\int \frac{w}{6} e^w \frac{dw}{12} = \frac{1}{6 \cdot 12} [w e^w - \int e^w dw]$$

$$u = w \quad dv = e^w dw$$

$$du = dw \quad v = e^w$$

$$\frac{1}{6 \cdot 12} (w-1)e^w = \frac{1}{72} (6x^2-1)e^{6x^2} + C$$

10. Find the indefinite integral.

$$\int \frac{8x^2}{e^x} dx$$

$$u = 8x^2 \quad dv = e^{-x} dx$$

$$du = 16x dx \quad v = -e^{-x}$$

$$-8x^2 e^{-x} + 16 \int x e^{-x} dx$$

$$u = x \quad dv = e^{-x} dx$$

$$du = dx \quad v = -e^{-x}$$

$$-8x^2 e^{-x} + 16 \left[-x e^{-x} - \int -e^{-x} dx \right]$$

$$-8x^2 e^{-x} - 16x e^{-x} - 16 e^{-x} + C$$

$$-\frac{8}{e^x} (x^2 + 2x + 2) + C$$

11. Find the indefinite integral.

$$\int x^3 \ln x dx$$

$$u = \ln x \quad dv = x^3 dx$$

$$du = \frac{dx}{x}$$

$$v = \frac{x^4}{4}$$

$$\frac{x^4}{4} \ln(x) - \int \frac{x^4}{4} \frac{dx}{x} = \frac{x^4}{4} \ln(x) - \frac{x^4}{16}$$

$$= \frac{x^4}{16} (4 \ln(x) - 1) + C$$

12. Find the indefinite integral.

$$\int \frac{3(\ln x)^2}{x^2} dx = \frac{-3(\ln x)^2}{x} - \int \frac{-6 \ln x}{x^2} dx = \frac{-3(\ln x)^2}{x} + 6 \left[-\frac{\ln x}{x} - \int \frac{-dx}{x^2} \right]$$

$$u = (\ln x)^2 \quad dv = 3x^{-2} dx \quad \hat{u} = \ln x \quad dv = x^{-2} dx$$

$$du = 2 \frac{\ln x}{x} dx \quad v = -3x^{-1} \quad d\hat{u} = \frac{dx}{x} \quad v = -x^{-1}$$

$$= \frac{-3(\ln x)^2}{x} - \frac{6 \ln x}{x} - \frac{6}{x} + c$$

13. Find the indefinite integral.

$$\int u \ln(u+6) du \quad \hat{u} = \ln x \quad dv = (x-6) dx \quad \left(\frac{x^2}{2} - 6x \right) \ln x - \int \left(\frac{x^2}{2} - 6x \right) \frac{dx}{x}$$

$$x = u+6 \quad d\hat{u} = \frac{dx}{x} \quad v = \frac{x^2}{2} - 6x \quad \left(\frac{x^2}{2} - 6x \right) \ln x + \int \left(6 - \frac{x}{2} \right) dx$$

$$dx = du \quad \int (x-6) \ln x dx = \left(\frac{x^2}{2} - 6x \right) \ln x + 6x - \frac{x^2}{4} + c$$

14. Find the indefinite integral.

$$\int \frac{\ln q}{2q^3} dq = \left[\frac{(u+6)^2}{2} - 6(u+6) \right] \ln(u+6) + 6u - \frac{(u+6)^2}{4} + c$$

$$u = \ln q \quad dv = \frac{q^{-3}}{2} dq \quad \frac{u^2 - 36}{2} \ln(u+6) + 6u - \frac{(u+6)^2}{4} + c$$

$$du = \frac{dq}{q} \quad v = -\frac{q^{-2}}{4} \quad -\frac{\ln q}{4q^2} + \int \frac{dq}{4q^3} = -\frac{\ln q}{4q^2} - \frac{1}{8q^2} + c = \frac{-1}{8q^2} (2 \ln q + 1) + c$$

15. Find the indefinite integral.

$$\int v \sqrt{2v-9} dv \quad v = \frac{u+9}{2} \quad \frac{1}{2} \int (u+9) \sqrt{u} \frac{du}{2} = \frac{1}{4} \int (u^{\frac{3}{2}} + 9u^{\frac{1}{2}}) du$$

$$u = 2v-9 \quad \frac{1}{4} \left[\frac{u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{9u^{\frac{3}{2}}}{\frac{3}{2}} \right] = \frac{u^{\frac{3}{2}}}{2} \left[\frac{u}{5} + 3 \right] + c = \frac{(2v-9)^{\frac{3}{2}}}{2} \left(\frac{2v-9}{5} + 3 \right)$$

$$du = 2dv \quad \frac{du}{2} = dv \quad = \frac{(2v-9)^{\frac{3}{2}}}{5} (v+3) + c$$

16. Find the indefinite integral.

$$\int \frac{t}{\sqrt{1+3t}} dt \quad \int \frac{u-1}{3} u^{-\frac{1}{2}} \frac{du}{3} = \frac{1}{9} \int (u^{\frac{1}{2}} - u^{-\frac{1}{2}}) du$$

$$u = 1+3t \quad = \frac{1}{9} \left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}} - \frac{u^{\frac{1}{2}}}{\frac{1}{2}} \right) = \frac{2\sqrt{u}}{27} (u-3) + c$$

$$du = 3dt \quad = \frac{2\sqrt{1+3t}}{27} (3t-2) + c$$

$$\frac{u-1}{3} = t \quad = \frac{(2v-9)^{\frac{3}{2}}}{5} (v+3) + c$$

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17. Find the indefinite integral.

$$f(x) = \int e^{-7n} \sin 5n \, dn$$

$$u = \sin 5n \quad dv = e^{-7n} \, dn$$

$$du = 5 \cos 5n \, dn \quad v = \frac{e^{-7n}}{-7}$$

$$-\frac{(\sin 5n) e^{-7n}}{7} + \frac{5}{7} \int e^{-7n} \cos 5n \, dn$$

$$\int e^{-7n} \cos 5n \, dn \quad u = \cos 5n \quad dv = e^{-7n} \, dn$$

$$du = -5 \sin 5n \, dn \quad v = \frac{e^{-7n}}{-7}$$

$$-\frac{\cos 5n e^{-7n}}{7} - \frac{5}{7} \int e^{-7n} \sin 5n \, dn$$

$$\frac{74}{49} f(x) = \left(-\frac{\sin(5n)}{7} - \frac{5 \cos 5n}{7^2} \right) e^{-7n}$$

$$f(x) = \frac{-7 \sin(5n) - 5 \cos(5n)}{74} e^{-7n} + c$$

18. Find the indefinite integral.

$$f(x) = \int e^{9u} \cos 6u \, du$$

$$w = e^{9u} \quad dv = \cos 6u \, du$$

$$dw = 9e^{9u} \, du \quad v = \frac{\sin 6u}{6}$$

$$\int e^{9u} \sin 6u \, du$$

$$w = e^{9u} \quad dv = \sin 6u \, du$$

$$dw = 9e^{9u} \, du \quad v = -\frac{\cos 6u}{6}$$

$$-\frac{e^{9u} \cos 6u}{6} + \frac{9}{6} \int e^{9u} \cos 6u \, du$$

$$f(x) = \left(\frac{1}{6} \sin 6u + \frac{9}{36} \cos 6u\right) e^{9u} - \left(\frac{9}{6}\right)^2 f(x)$$

$$(36+81)f(x) = (6 \sin 6u + 9 \cos 6u) e^{9u}$$

$$f(x) = \frac{e^{9u} (6 \sin 6u + 9 \cos 6u)}{9 \cdot 13}$$

$$= \frac{e^{9u} (2 \sin 6u + 3 \cos 6u)}{39} + C$$

19. Find the definite integral.

$$\int_0^{\pi/5} x \cos 5x \, dx$$

$$u = x \quad dv = \cos 5x \, dx$$

$$du = dx \quad v = \frac{\sin 5x}{5}$$

$$\frac{x \sin 5x}{5} - \int \frac{\sin 5x}{5} \, dx = \left[\frac{x \sin 5x}{5} + \frac{\cos 5x}{5^2} \right]_0^{\pi/5}$$

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$$\rightarrow \frac{\pi \sin \pi}{25} + \frac{\cos \pi}{25} - \left(\frac{0 \sin 0}{5} + \frac{\cos 0}{25} \right) = -\frac{1}{25} - \frac{1}{25} = -\frac{2}{25}$$

20. Find the definite integral.

$$\int_1^3 x^3 \ln x \, dx$$

$$u = \ln x \quad dv = x^3 \, dx$$

$$du = \frac{dx}{x} \quad v = \frac{x^4}{4}$$

$$\frac{x^4}{4} \ln x - \int \frac{x^4}{4} \frac{dx}{x} = \frac{x^4}{4} \ln x - \frac{1}{4} \int x^3 \, dx$$

$$= \left[\frac{x^4}{4} \left(\ln x - \frac{1}{4} \right) \right]_1^3 = \frac{81}{4} \left(\ln 3 - \frac{1}{4} \right) - \left(-\frac{1}{16} \right)$$

$$= \frac{81}{4} \ln 3 - \frac{20}{4} = \frac{81 \ln 3 - 20}{4}$$

21. Find the indefinite integral.

$$\int \cos x \sin^4 x \, dx$$

$$u = \sin x$$

$$du = \cos x \, dx$$

$$= \int u^4 \, du = \frac{u^5}{5} + C = \frac{\sin^5 x}{5} + C$$

22. Find the indefinite integral.

$$\int \sin x \cos^3 x \, dx$$

$$u = \cos x$$

$$du = -\sin x \, dx$$

$$= -\int u^3 \, du = -\frac{u^4}{4} + C = -\frac{\cos^4 x}{4} + C$$

23. Find the indefinite integral.

$$\int \sin^3 4x \cos^4 4x \, dx$$

$$= \int (1 - \cos^2 4x) \cos^4 4x \sin 4x \, dx$$

$$u = \cos 4x$$

$$du = -4 \sin 4x \, dx$$

$$-\frac{1}{4} \int (1 - u^2) u^4 \, du = \frac{1}{4} \int (u^6 - u^4) \, du$$

$$= \frac{1}{4} \left(\frac{u^7}{7} - \frac{u^5}{5} \right) + C = \frac{1}{4} \left(\frac{\cos^7 4x}{7} - \frac{\cos^5 4x}{5} \right) + C$$

24. Find the indefinite integral.

$$\int \cos^3 4x \sin^2 4x dx$$

$$= \int (1 - \sin^2 4x) \sin^2 4x \cos 4x dx$$

$$u = \sin 4x \quad du = 4 \cos 4x dx$$

$$\frac{1}{4} \int u^2 - u^4 du$$

$$= \frac{1}{4} \left(\frac{u^3}{3} - \frac{u^5}{5} \right) + C$$

$$= \frac{1}{4} \left(\frac{\sin^3 4x}{3} - \frac{\sin^5 4x}{5} \right) + C$$

25. Find the indefinite integral.

$$\int \cos^3 2x dx$$

$$\int (1 - \sin^2 2x) \cos 2x dx$$

$$u = \sin 2x \quad du = 2 \cos 2x dx$$

$$\frac{1}{2} \int (1 - u^2) du$$

$$= \frac{1}{2} \left(u - \frac{u^3}{3} \right) = \frac{1}{2} \left(\sin 2x - \frac{\sin^3 2x}{3} \right) + C$$

26. Find the indefinite integral.

$$\int \cos^4 5x dx$$

$$\int \left(\frac{1 + \cos 10x}{2} \right)^2 dx$$

$$\frac{1}{4} \int (1 + 2 \cos 10x + \cos^2 10x) dx$$

$$\frac{1}{4} \left(x + \frac{2 \sin 10x}{10} \right) + \frac{1}{4} \int \cos^2 10x dx$$

$$\frac{1}{4} \left(\frac{3x}{2} + \frac{\sin 10x}{5} + \frac{\sin 20x}{40} \right) + C$$

27. Find the indefinite integral.

$$\int \sin^3 \frac{x}{5} dx$$

$$\int (1 - \cos^2 \frac{x}{5}) \sin \frac{x}{5} dx$$

$$u = \cos \frac{x}{5}$$

$$du = -\frac{1}{5} \sin \frac{x}{5} dx$$

$$-5 \int (1 - u^2) du$$

$$= -5 \left(u - \frac{u^3}{3} \right) = -5 \left(\cos \frac{x}{5} - \frac{\cos^3 \frac{x}{5}}{3} \right) + C$$

$$= \frac{5}{3} \cos^3 \frac{x}{5} - 5 \cos \frac{x}{5} + C$$

28. Find the indefinite integral.

$$\int \sin^2 2x dx$$

$$\frac{1}{2} \int (1 - \cos 4x) dx = \frac{1}{2} \left(x - \frac{1}{4} \sin 4x \right) + C = \frac{4x - \sin 4x}{8} + C$$

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29. Find the indefinite integral.

$$\int \sin^3 2\theta \sqrt{\cos 2\theta} d\theta$$

$$\int (1 - \cos^2 2\theta) \sin 2\theta \sqrt{\cos 2\theta} d\theta = -\frac{1}{2} \int (1 - u^2) \sqrt{u} du$$

$$u = \cos 2\theta$$

$$du = -2 \sin 2\theta d\theta$$

$$= \frac{1}{2} \int u^{\frac{5}{2}} - u^{\frac{1}{2}} du$$

$$= \frac{1}{2} \left(\frac{u^{\frac{7}{2}}}{\frac{7}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$

$$= \left(\frac{\cos^{\frac{7}{2}} 2\theta}{7} - \frac{\cos^{\frac{3}{2}} 2\theta}{3} \right) \sqrt{\cos 2\theta} + C$$

30. Find the indefinite integral.

$$\int \frac{\cos^3 \theta}{\sqrt{\sin \theta}} d\theta$$

$$\int \frac{1 - \sin^2 \theta}{\sqrt{\sin \theta}} \cos \theta d\theta$$

$$u = \sin \theta$$

$$du = \cos \theta d\theta$$

$$\int \frac{1 - u^2}{\sqrt{u}} du$$

$$\int u^{-\frac{1}{2}} - u^{\frac{3}{2}} du$$

$$\frac{u^{\frac{1}{2}}}{\frac{1}{2}} - \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$2\sqrt{\sin \theta} - \frac{2}{5} \sin^2 \theta \sqrt{\sin \theta} + C$$

31. Find the indefinite integral.

$$\int \tan^3 7x dx$$

$$\int (\sec^2 7x - 1) \tan 7x dx = \frac{1}{7} \int u du - \int \tan 7x dx$$

$$u = \tan 7x$$

$$du = 7 \sec^2 7x dx$$

$$= \frac{u^2}{14} + \frac{1}{7} \ln |\cos 7x| = \frac{1}{14} (\tan^2 7x + 2 \ln |\cos 7x|) + C$$

32. Find the indefinite integral.

$$\int \tan^5 \left(\frac{x}{11} \right) dx$$

$$\int (\sec^2 \left(\frac{x}{11} \right) - 1)^2 \tan \left(\frac{x}{11} \right) dx$$

$$\int (\sec^4 \left(\frac{x}{11} \right) - 2 \sec^2 \left(\frac{x}{11} \right) - 1) \tan \left(\frac{x}{11} \right) dx$$

$$\int \left[\left(\tan^2 \frac{x}{11} + 1 \right) - 2 \right] \sec^2 \frac{x}{11} - 1 \tan \frac{x}{11} dx$$

$$u = \tan \frac{x}{11}$$

$$du = \frac{1}{11} \sec^2 \frac{x}{11} dx$$

$$11 \int (u^3 - u) du - \int \tan \frac{x}{11} dx$$

$$11 \left(\frac{\tan^4 \left(\frac{x}{11} \right)}{4} - \frac{11 \tan^2 \frac{x}{11}}{2} + 11 \ln |\cos \left(\frac{x}{11} \right)| \right) + C$$

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33. Find the indefinite integral.

$$\int \sec^4 2x dx$$

$$\int (\tan^2 2x + 1) \sec^2 2x dx$$

$$u = \tan 2x \quad du = 2 \sec^2 2x dx$$

$$\frac{1}{2} \int (u^2 + 1) du = \frac{1}{2} \left(\frac{u^3}{3} + u \right) + C$$

$$= \frac{\tan^3 2x}{6} + \frac{\tan 2x}{2} + C$$

34. Find the indefinite integral.

$$\int \sec^6 4x dx$$

$$\int (\tan^2 4x + 1)^2 \sec^2 4x dx$$

$$u = \tan 4x$$

$$du = 4 \sec^2 4x dx$$

$$= \frac{1}{4} \int (u^2 + 1)^2 du = \frac{1}{4} \int (u^4 + 2u^2 + 1) du$$

$$= \frac{1}{4} \left(\frac{u^5}{5} + \frac{2u^3}{3} + u \right) = \frac{1}{4} \left(\frac{\tan^5 4x}{5} + \frac{2 \tan^3 4x}{3} + \tan 4x \right) + C$$

35. Find the indefinite integral.

$$\int \tan^3 \left(\frac{\pi x}{7} \right) \sec^2 \left(\frac{\pi x}{7} \right) dx$$

$$u = \tan \left(\frac{\pi x}{7} \right)$$

$$du = \frac{\pi}{7} \sec^2 \frac{\pi x}{7} dx$$

$$\frac{7}{\pi} \int u^3 du = \frac{7}{\pi} \frac{u^4}{4} + C$$

$$\frac{7}{4\pi} \tan^4 \left(\frac{\pi x}{7} \right) + C$$

36. Find the indefinite integral.

$$\int \sec^5 3x \tan 3x dx$$

$$\frac{1}{3} \int u^4 du = \frac{u^5}{15} + C = \frac{\sec^5 3x}{15} + C$$

$$u = \sec 3x$$

$$du = 3 \sec 3x \tan 3x dx$$

37. Find the arc length of the graph of the function $y = \frac{2}{3}x^{\frac{3}{2}} + 2$ over the interval $[0, 12]$.

$$y' = x^{\frac{1}{2}}$$

$$S = \int_0^{12} \sqrt{1+(x^{\frac{1}{2}})^2} dx \rightarrow \int_0^{12} \sqrt{1+x} dx$$

$$u = x+1$$

$$du = dx$$

$$= \int_{x=0}^{x=12} u^{\frac{1}{2}} du = \left[\frac{2}{3} u^{\frac{3}{2}} \right]_1^{13} = \frac{2}{3} (13^{\frac{3}{2}} - 1)$$

38. Find the arc length of the graph of the function $y = 18x^{\frac{3}{2}} + 6$ over the interval $[0, 7]$.

$$y' = 27x^{\frac{1}{2}}$$

$$S = \int_0^7 \sqrt{1+27^2 x} dx$$

$$u = 1+27^2 x$$

$$du = 27^2 dx$$

$$\frac{1}{27^2} \int_{x=0}^{x=7} u^{\frac{1}{2}} du = \left[\frac{2}{3 \cdot 27^2} u^{\frac{3}{2}} \right]_1^{1+7 \cdot 27^2}$$

$$= \frac{2}{3 \cdot 27^2} \left((1+7 \cdot 27^2)^{\frac{3}{2}} - 1 \right)$$

39. Find the arc length of the graph of the function $y = \frac{3}{2}x^{\frac{2}{3}} + 10$ over the interval $[1, 512]$.

$$y' = x^{-\frac{1}{3}}$$

$$S = \int_1^{512} \sqrt{1+x^{-\frac{2}{3}}} dx$$

$$u = x^{\frac{2}{3}} + 1$$

$$du = \frac{2}{3} x^{-\frac{1}{3}} dx$$

$$\frac{3}{2} \int_{x=1}^{x=512} \sqrt{u} du = \left[\frac{3}{2} u^{\frac{3}{2}} \right]_{u=2}^{u=65}$$

$$= \frac{3}{2} (65^{\frac{3}{2}} - 2^{\frac{3}{2}})$$

40. Find the arc length of the graph of the function $y = \frac{x^8}{16} + \frac{1}{12x^6}$ over the interval $[1, 2]$.

$$y' = \frac{x^7}{2} + \frac{1}{2x^7}$$

$$y'' = \frac{x^6}{4} - \frac{1}{2} + \frac{1}{4x^8}$$

$$S = \int_1^2 \sqrt{1 + \frac{x^{14}}{4} - \frac{1}{2} + \frac{1}{4x^{14}}} dx = \int_1^2 \left(\frac{x^7}{2} + \frac{1}{2x^7} \right) dx$$

$$= \left[\frac{x^8}{16} - \frac{1}{12x^6} \right]_1^2 = \frac{256}{16} - \frac{1}{12 \cdot 64} - \left(\frac{1}{16} - \frac{1}{12} \right)$$

41. Find the arc length of the graph of the function $x = \frac{1}{3}(y^2 + 2)^{\frac{3}{2}}$ over the interval $0 \leq y \leq 14$.

$$\frac{dx}{dy} = \frac{1}{2} (y^2 + 2)^{\frac{1}{2}} \cdot 2y$$

$$\left(\frac{dx}{dy} \right)^2 = (y^2 + 2)y^2 = y^4 + 2y^2$$

$$S = \int_0^{14} \sqrt{1 + 2y^2 + y^4} dy$$

$$\int_0^{14} (y^2 + 1) dy = \left[\frac{y^3}{3} + y \right]_0^{14}$$

$$\frac{14^3}{3} + 14$$

42. Find the arc length of the graph of the function $x = \frac{1}{3}(y-3)\sqrt{y}$ over the interval $1 \leq y \leq 9$.

$$\frac{dx}{dy} = \frac{1}{2} y^{\frac{1}{2}} - \frac{1}{2} y^{-\frac{1}{2}}$$

$$\left(\frac{dx}{dy}\right)^2 = \frac{y}{4} - \frac{1}{2} + \frac{y^{-1}}{4}$$

$$s = \int_1^9 \sqrt{1 + \frac{y}{4} - \frac{1}{2} + \frac{y^{-1}}{4}} dy$$

$$= \int_1^9 \sqrt{\frac{y}{4} + \frac{1}{2} + \frac{y^{-1}}{4}} dy = \int_1^9 \left(\frac{y^{\frac{1}{2}}}{2} + \frac{1}{2} y^{-\frac{1}{2}} \right) dy$$

$$\left[\frac{1}{2} \left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}} + \frac{y^{\frac{1}{2}}}{\frac{1}{2}} \right) \right]_1^9 = \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} + y^{\frac{1}{2}} \right]_1^9 = \left(\frac{27}{\frac{3}{2}} + 3 \right) - \left(\frac{1}{\frac{3}{2}} + 1 \right)$$

$$= 12 - 1 - \frac{1}{3} = 10 + \frac{2}{3}$$