

THEOREM 9.8 Arc Length in Parametric Form

If a smooth curve C is given by $x = f(t)$ and $y = g(t)$ such that C does not intersect itself on the interval $a \leq t \leq b$ (except possibly at the endpoints), then the arc length of C over the interval is given by

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2} dt.$$

THEOREM 9.9 Area of a Surface of Revolution

If a smooth curve C given by $x = f(t)$ and $y = g(t)$ does not cross itself on an interval $a \leq t \leq b$, then the area S of the surface of revolution formed by revolving C about the coordinate axes is given by the following.

1. $S = 2\pi \int_a^b g(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$ Revolution about the x -axis: $g(t) \geq 0$
2. $S = 2\pi \int_a^b f(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$ Revolution about the y -axis: $f(t) \geq 0$

THEOREM 9.11 Slope in Polar Form

If f is a differentiable function of θ , then the *slope* of the tangent line to the graph of $r = f(\theta)$ at the point (r, θ) is

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta) \cos \theta + f'(\theta) \sin \theta}{-f(\theta) \sin \theta + f'(\theta) \cos \theta}$$

provided that $dx/d\theta \neq 0$ at (r, θ) . (See Figure 9.44.)

THEOREM 9.12 Tangent Lines at the Pole

If $f(\alpha) = 0$ and $f'(\alpha) \neq 0$, then the line $\theta = \alpha$ is tangent at the pole to the graph of $r = f(\theta)$.

THEOREM 9.13 Area in Polar Coordinates

If f is continuous and nonnegative on the interval $[\alpha, \beta]$, $0 < \beta - \alpha \leq 2\pi$, then the area of the region bounded by the graph of $r = f(\theta)$ between the radial lines $\theta = \alpha$ and $\theta = \beta$ is given by

THEOREM 9.14 Arc Length of a Polar Curve

Let f be a function whose derivative is continuous on an interval $\alpha \leq \theta \leq \beta$. The length of the graph of $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ is

$$s = \int_{\alpha}^{\beta} \sqrt{[f(\theta)]^2 + [f'(\theta)]^2} d\theta = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

THEOREM 9.15 Area of a Surface of Revolution

Let f be a function whose derivative is continuous on an interval $\alpha \leq \theta \leq \beta$. The area of the surface formed by revolving the graph of $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ about the indicated line is as follows.

1. $S = 2\pi \int_{\alpha}^{\beta} f(\theta) \sin \theta \sqrt{[f(\theta)]^2 + [f'(\theta)]^2} d\theta$ About the polar axis

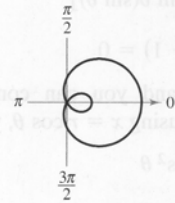
2. $S = 2\pi \int_{\alpha}^{\beta} f(\theta) \cos \theta \sqrt{[f(\theta)]^2 + [f'(\theta)]^2} d\theta$ About the line $\theta = \frac{\pi}{2}$

Limaçons

$$r = a \pm b \cos \theta$$

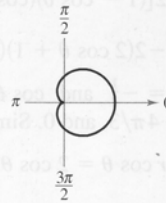
$$r = a \pm b \sin \theta$$

$$(a > 0, b > 0)$$



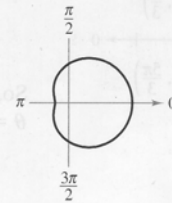
$$\frac{a}{b} < 1$$

Limaçon with
inner loop



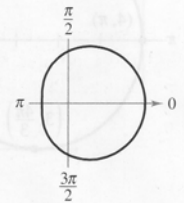
$$\frac{a}{b} = 1$$

Cardioid
(heart-shaped)



$$1 < \frac{a}{b} < 2$$

Dimpled limaçon



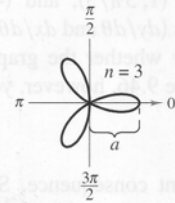
$$\frac{a}{b} \geq 2$$

Convex limaçon

Rose Curves

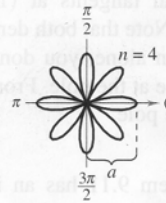
n petals if n is odd

$2n$ petals if n is even
($n \geq 2$)



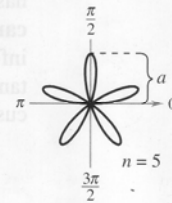
$$r = a \cos n\theta$$

Rose curve



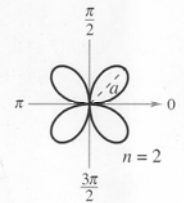
$$r = a \cos n\theta$$

Rose curve



$$r = a \sin n\theta$$

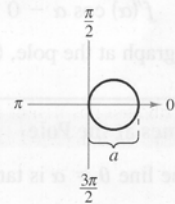
Rose curve



$$r = a \sin n\theta$$

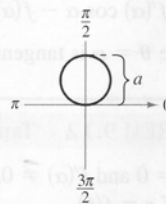
Rose curve

Circles and Lemniscates



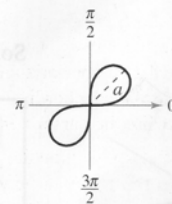
$$r = a \cos \theta$$

Circle



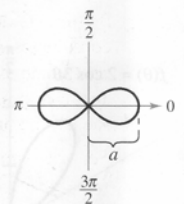
$$r = a \sin \theta$$

Circle



$$r^2 = a^2 \sin 2\theta$$

Lemniscate



$$r^2 = a^2 \cos 2\theta$$

Lemniscate