M250 Practice 6.1-6.3 & 7.1
P. Staley Larson 8th

1. Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = -3x^3 + x$$

$\int dy = \int (-3x^3 + x)\,dx$

$y = -\frac{3x^4}{4} + \frac{x^2}{2} + c$

2. Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = -8x^2 + x$$

$dy = -8x^2 + x\ dx$

$\int dy = \int -8x^2 + x\,dx$

$y = -\frac{8x^3}{3} + \frac{x^2}{2} + c$

3. Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = x\sqrt{x-2}$$

$dy = x\sqrt{x-2}\,dx$

$\int dy = \int x\sqrt{x-2}\,dx$

Let $u = x-2$, $du = dx$

$y = \int u+2\sqrt{u}\,du$

$y = \frac{2u^{5/2}}{5} + \frac{4}{3}u^{3/2} + c$

$y = \frac{2}{5}(x-2)^{5/2} + \frac{4}{3}(x-2)^{3/2} + c$

4. Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = x\sqrt{13-x^2}$$

$dy = x\sqrt{13-x^2}\,dx$

$u = 13-x^2$, $du = -2x\,dx$

$\int dy = -\frac{1}{2}\int \sqrt{u}\,du$

$y = -\frac{1}{3}u^{\frac{3}{2}} + c$

$y = -\frac{(13-x^2)^{\frac{3}{2}}}{3} + c$

5. Use integration to find a general solution of the differential equation.

$$\frac{dy}{dx} = x^5 e^{x^6}$$

$dy = x^5 e^{x^6}\,dx$

$u = x^6$, $du = 6x^5\,dx$

$\int dy = \int \frac{1}{6} e^u\,du$

$y = \frac{1}{6}e^u + c$

$y = \frac{1}{6}e^{x^6} + c$

6. Select from the choices below the slope field for the differential equation.

$\frac{dy}{dx} = \cos(5x)$ OMIT

7. Use Euler's Method to make a table of values for the approximate solution of the following differential equation with specified initial value. Use 10 steps of size 0.05.

$y' = -4x + 3y$, $y(0) = 5$.

8. Solve the differential equation.

$\frac{dy}{dx} = y + 5$

$\frac{dy}{y+5} = dx$

$\int \frac{dy}{y+5} = \int dx$

$\ln(y+5) = x + \hat{c}$

$y + 5 = e^{x+\hat{c}}$

$y = ce^{x} - 5$

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9. Solve the differential equation.

$\frac{dy}{dx} = -x - 4$

$dy = (-x - 4)dx$

$\int dy = -\int x + 4 dx$

$y = -\frac{x^2}{2} - 4x + c$

10. Solve the differential equation.

$y' = \frac{\sqrt{x}}{8y}$

$8y\,dy = \sqrt{x}\,dx$

$\int 8y\,dy = \int \sqrt{x}\,dx$

$4y^2 = \frac{2}{3}x^{3/2} + \hat{c}$

$6y^2 - x^{3/2} = c$

11. Solve the differential equation.

$y' = \sqrt{19xy}$

$\frac{dy}{y} = \sqrt{19x}\,dx$

$\int \frac{dy}{y} = \int \sqrt{19x}\,dx$

$\ln y = \sqrt{19}\,\frac{2}{3}x^{\frac{3}{2}} + \hat{c}$

$y = Ce^{\frac{2}{3}\sqrt{19}x^{\frac{3}{2}}}$

12. Solve the differential equation.

$y' = \frac{8x}{y}$

$y\,dy = 8x\,dx$

$\int y\,dy = \int 8x\,dx$

$\frac{y^2}{2} = 4x^2 + \hat{c}$

$8x^2 - y^2 = c$

13. Solve the differential equation.

$y' = x(8+y)$

$\frac{dy}{8+y} = xd$

$\int \frac{dy}{y+8} = x\,dx$

$\ln(y+8) = \frac{x^2}{2} + \hat{c}$

$y+8 = ce^{\frac{x^2}{2}}$

$y = ce^{\frac{x^2}{2}} - 8$

14. Write and solve the differential equation that models the following verbal statement:

The rate of change of M with respect to s is proportional to $200 - s$.

$\frac{dM}{ds} = k(200-s)$

$\int dM = \int k(200-s)\,ds$

$M = k\left(200s - \frac{s^2}{2}\right) + C$

15. Write and solve the differential equation that models the following verbal statement. Evaluate the solution at the specified value of the independent variable, rounding your answer to four decimal places:

The rate of change of V is proportional to V. When $r = 0$, $V = 64$ and when $r = 2$, $V = 72$. What is the value of V when $r = 9$?

$\frac{dV}{dr} = kV$

$\int \frac{dV}{V} = \int k\,dr$

$\ln V = kr + \hat{c}$

$V = e^{kr+\hat{c}}$

$\boxed{V = Ce^{kr}}$

$r=0,\ V=64$

$\Rightarrow C = 64$

$V = 64e^{kr}$

$r=2,\ V=72 \Rightarrow 72 = 64e^{2k}$

$\Rightarrow r = \frac{1}{2}\ln\left(\frac{72}{64}\right)$

$\Rightarrow V = 64\left(\frac{72}{64}\right)^{\frac{r}{2}} = 64\left(\frac{9}{8}\right)^{\frac{r}{2}}$

when $r = 9$, $V = 64\left(\frac{9}{8}\right)^{4.5} = 108.75$

16. The isotope ^{239}Pu has a half-life of 24,100 years. Given an intial amount of 6 grams of the isotope, how many grams will remain after 1,000 years? After 10,000 years? Round your answers to four decimal places.

17. The isotope ^{239}Pu has a half-life of 24,100 years. After 1,000 years, a sample of the isotope is reduced to 2.8 grams. What was the initial size of the sample (in grams)? How much will remain after 10,000 years (i.e., after another 9,000 years)? Round your answers to four decimal places.

$y = y_0\left(\frac{1}{2}\right)^{\frac{t}{24100}}$ $\quad$ $y_0 = 2.8 * 2^{\frac{1}{24.1}} \approx 2.8817$

$2.8 = y_0\left(\frac{1}{2}\right)^{\frac{1000}{24100}}$ $\quad$ $y_{10000} = 2.8 * 2^{\frac{1}{24.1}}\left(\frac{1}{2}\right)^{\frac{10000}{24100}} = 2.8\left(\frac{1}{2}\right)^{\frac{10}{24.1} - \frac{1}{24.1}} = 2.8\left(\frac{1}{2}\right)^{\frac{9}{24.1}}$

≈ 2.1614

18. The isotope ^{14}C has a half-life of 5,715 years. After 15,000 years, a sample of the isotope is reduced 0.9 grams. What was the initial size of the sample (in grams)? How large was the sample after the first 1,500 years? Round your answers to four decimal places.

19. The number of bacteria in a culture is increasing according to the law of exponential growth. After 2 hours there are 185 bacteria in the culture and after 4 hours there are 450 bacteria in the culture. Answer the following questions, rounding numerical answers to four decimal places.

$y = y_0 e^{kt}$ $\quad$ $(2, 185) \Rightarrow 185 = y_0 e^{2k}$ $\quad$ $(4, 450) \Rightarrow 450 = y_0 e^{4k}$

$\frac{450}{185} = \frac{y_0 e^{4k}}{y_0 e^{2k}}$ $\quad$ $\Rightarrow \frac{450}{185} = e^{2k}$ $\quad$ $\Rightarrow 185 = y_0 \frac{450}{185}$ so $y_0 = \frac{185^2}{450}$

(i) Find the initial population.

(ii) Write an exponential growth model for the bacteria population. Let t represent time in hours.

$y = \frac{185^2}{450} e^{kt}$, $185 = \frac{185^2}{450} e^{2k} \Rightarrow \frac{450}{185} = e^{2k} \Rightarrow k = \frac{1}{2}\ln\frac{450}{185}$ so $y = \frac{185^2}{450}\left(\frac{450}{185}\right)^{\frac{t}{2}}$

(iii) Use the model to determine the number of bacteria after 8 hours.

$y(8) = \frac{185^2}{450}\left(\frac{450}{185}\right)^4 = \frac{450^3}{185^2} \approx 2663$

(iv) After how many hours will the bacteria count be 20,000?

$20000 = \frac{185^2}{450}\left(\frac{450}{185}\right)^{\frac{t}{2}}$

$\frac{450}{185^2} \cdot 20000 = \left(\frac{450}{185}\right)^{\frac{t}{2}}$ $\quad$ $\rightarrow \ln \frac{450}{185^2} 2000 = \frac{t}{2} \ln \frac{450}{185}$

$t = \frac{2\ln\left(\frac{450}{185^2}\right)20000}{\ln\frac{450}{185}} = 12.537$ hours

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20. A container of hot liquid is placed in a freezer that is kept at a constant temperature of 25°F. The initial temperature' of the liquid is 190°F. After 7 minutes, the liquid´s temperature is 62°F. How much longer will it take for its temperature to decrease to 31°F? Round your answer to two decimal places.

$y' = k(y-25) \Rightarrow \ln(y-25) = kt + \hat{c}$

$\dfrac{dy}{y-25} = k\,dt \Rightarrow y = ce^{kt} + 25$

$y(0) = 190 \Rightarrow 190 = c + 25$

$\Rightarrow y = 165e^{kt} + 25$

$y(7) = 62 \Rightarrow$

$62 = 165e^{7k} + 25$

$\Rightarrow \dfrac{37}{165} = e^{7k} \Rightarrow k = \dfrac{1}{7}\ln\dfrac{37}{165}$

$\Rightarrow y = 165\left(\dfrac{37}{165}\right)^{\frac{t}{7}} + 25$

$y = 31 \Rightarrow$

$31 = 165\left(\dfrac{37}{165}\right)^{\frac{t}{7}} + 25$

$\Rightarrow \ln\dfrac{6}{165} = \dfrac{t}{7}\ln\left(\dfrac{37}{165}\right)$

$\Rightarrow t = 15.52$ minutes

21. Find the general solution of the differential equation.

$$\frac{dy}{dx} = \frac{-7x}{y}$$

$y\,dy = -7x\,dx$

$\int y\,dy = -7\int x\,dx$

$\dfrac{y^2}{2} = \dfrac{-7x^2}{2} + \hat{c}$

$y^2 + 7x^2 = c$

22. Find the general solution of the differential equation.

$$\frac{dy}{dx} = \frac{x^2+2}{-5y^2}$$

$-5y^2\,dy = (x^2+2)\,dx$

$-5\int y^2\,dy = \int (x^2+2)\,dx$

$\Rightarrow -\dfrac{5y^3}{3} = \dfrac{x^3}{3} + 2x + \hat{c}$

$x^3 + 5y^3 + 6x = c$

23. Find the general solution of the differential equation.

$$(-6+x)y' = -2y$$

$\dfrac{dy}{-2y} = \dfrac{dx}{x-6}$

$\int \dfrac{dy}{-2y} = \int \dfrac{dx}{x-6}$

$-\dfrac{1}{2}\ln y = \ln(x-6) + \hat{c}$

$\ln y = -2\ln(x-6) + \hat{c}$

$\ln y = \ln\left(\dfrac{c}{(x-6)^2}\right)$

$\Rightarrow y = \dfrac{c}{(x-6)^2}$

24. Find the general solution of the differential equation.

$$x^{15}y' = y$$

25. Find the general solution of the differential equation.

$yy' = \tan(3x)$

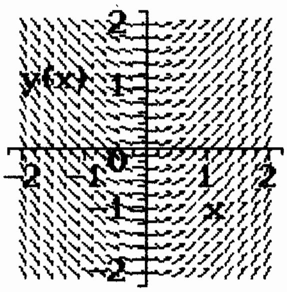
$\int y\,dy = \int \tan(3x)\,dx$

$\frac{y^2}{2} = \frac{1}{3}\ln|\sec(3x)| + c$

$y = \pm\sqrt{\frac{2}{3}\ln|\sec(3x)| + c}$

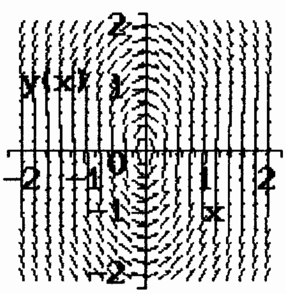
26. Sketch a few solutions of the differential equation on the slope field and then find the general solution analytically.

$$\frac{dy}{dx} = x$$



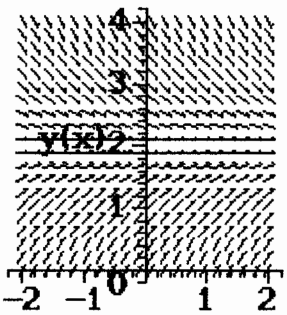
27. Sketch a few solutions of the differential equation on the slope field and then find the general solution analytically.

$$\frac{dy}{dx} = \frac{-4x}{y}$$



28. Sketch a few solutions of the differential equation on the slope field and then find the general solution analytically.

$$\frac{dy}{dx} = 2 - y$$



29. Find the logistic equation that satisfies the following differential equation and initial condition.

$$\frac{dy}{dt} = 3.2y\left(1 - \frac{y}{19}\right), \ y(0) = 4$$

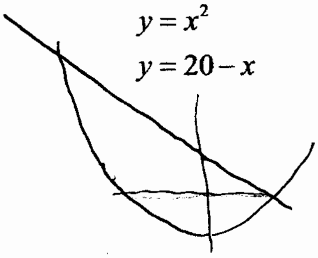
30. Find the area of the region bounded by the equations by integrating (i) with respect to x and (ii) with respect to y.

$$x = 16 - y^2$$
$$x = y - 4$$

31. Find the area of the region bounded by equations by integrating (i) with respect to x and (ii) with respect to y.

$y = x^2$

$y = 20 - x$



intersection:

$20-x=x^2$

$\Rightarrow x^2+x-20=0$

$(x+5)(x-4)=0$

$\Rightarrow$ interection at $(-5,25)$ and $(4,16)$

(i) $\int_{-5}^{4}(20-x)-x^2\,dx = \left[-\frac{x^3}{3}-\frac{x^2}{2}+20x\right]_{-5}^{4}$

$=\left(\frac{-64}{3}-\frac{16}{2}+80\right)-\left(\frac{125}{3}-\frac{25}{2}-100\right)$

$=180-63+\frac{9}{2}=121.5$

(ii) $\int_{0}^{16}\sqrt{y}--\sqrt{y}\,dy+\int_{16}^{25}20-y--\sqrt{y}\,dy$

$=\left[\frac{4}{3}y^{\frac{3}{2}}\right]_0^{16}+\left[20y-\frac{y^2}{2}+\frac{2}{3}y^{\frac{3}{2}}\right]_{16}^{25}=\frac{4\cdot 64}{3}+20\cdot 9+\frac{16^2-25^2}{2}+\frac{2}{3}(5^3-4^3)$

$=121.5$

32. Find the area of the region bounded by the graphs of the algebraic functions.

$f(x) = x^2 - 16x$

$g(x) = 0$

33. Find the area of the region bounded by the graphs of the algebraic functions.

$f(x) = x^2 + 24x + 144$

$g(x) = 14(x+12)$

34. Find the area of the region bounded by the graphs of the algebraic functions.

$f(x) = \sqrt[3]{x-13}$

$g(x) = x - 13$

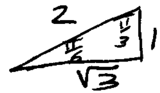
35. Find the area of the region bounded by the graphs of the algebraic functions.

$f(y) = y^2 + 9, \quad g(y) = 0, \quad y = -9, \quad y = 10$

36. Find the area of the region bounded by the graphs of the equations.

$$f(x) = \frac{14x}{x^2+1}, \quad y = 0, \quad 0 \leq x \leq 7$$

37. Find the area of the region bounded by the graphs of the equations.



$$f(x) = \sin(x), \quad g(x) = \cos(2x), \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{6}$$

note $g(x) \geq f(x)$ on $[-\frac{\pi}{2}, \frac{\pi}{6}]$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} \cos(2x) - \sin(x)\,dx$$

$$\left[\frac{1}{2}\sin(2x) + \cos(x)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{6}}$$

$$= \frac{1}{2}\sin\frac{\pi}{3} + \cos\left(\frac{\pi}{6}\right) + \frac{1}{2}\sin(-\pi) - \cos\left(-\frac{\pi}{2}\right)$$

$$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} + 0 + 0 = \frac{3}{4}\sqrt{3}$$

38. Find the area of the region bounded by the graphs of the equations.

$$f(x) = xe^{-x}, \quad y = 0, \quad 0 \leq x \leq 1$$