

M260

name: _____

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Point of View Survey

P1. The relationship among the sides of a right triangle expressed in the Pythagorean theorem is true because it has been

- a) verified by measurement.
- b) proven by logical argument.

Answer by selecting a response on the eight point scale, with “1” indicating the belief that it is only a), “7” only b), and answers between these indicating the appropriate mix. (For example “4” indicates equally a) and b).)

1 2 3 4 5 6 7 Neither

P2. If a mathematician asks for a mathematical proof for the Pythagorean theorem he is asking for

- a) verification by measurement.
- b) a purely logical argument.

answer using the same scale as in P1.:

1 2 3 4 5 6 7 Neither

Even and Odd functions.

Even functions are those for which $f(-x) = f(x)$, e.g. x^2 , $\cos(x)$, $x \sin(x)$, etc. Odd functions are those for which $f(-x) = -f(x)$, e.g. x^3 , $\sin(x)$, $\tan(x)$, etc. Try to decide whether the following statement is true or false:

“The derivative of every differentiable even function is an odd function.”

Circle one: true false

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Consider the statement:

“The derivative of every differentiable even function is an odd function.”

Below are five arguments purporting to demonstrate that this statement is true. Please assess, as a percentage, how personally convincing you find each argument.

A: _____ B: _____ C: _____ D: _____ E: _____

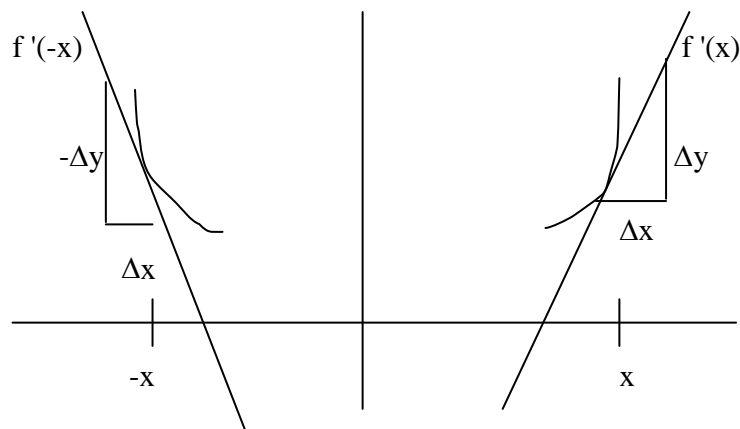
After you have finished the above question please assess the percentage a professor would assign each of the arguments if a “proof” of the original statement had been requested.

A: _____ B: _____ C: _____ D: _____ E: _____

Argument A:

Consider the following functions and their derivatives

$f(x) = x$ odd	$f(x) = x^2$ even	$f(x) = x^3$ odd
$f'(x) = 1$ even	$f'(x) = 2x$ odd	$f'(x) = 3x^2$ even
$f(x) = x^4$ even	$f(x) = x^5$ odd	$f(x) = x^6$ even
$f'(x) = 4x^3$ odd	$f'(x) = 5x^4$ even	$f'(x) = 6x^5$ odd

Note that for all the even functions, the derivative is odd. We could continue for all powers ($n = 7, 8, 9, \dots$), thus the claim is proved.**Argument B:**If f is an even function its graph is symmetric over the y axis. So the slope at any point x is the negative of the slope at $-x$. In other words $f'(-x) = -f'(x)$, which means the derivative of the function is odd.

Argument C:

Want to show if $f(x) = f(-x)$, then $-f'(x) = f'(-x)$.

$$f'(-x) = \lim_{h \rightarrow 0} \frac{f(-x+h) - f(-x)}{h} \quad (\text{by the definition of derivative})$$

$$= \lim_{h \rightarrow 0} \frac{f(x-h) - f(x)}{h} \quad (\text{since } f \text{ is even}).$$

Let $t = -h$.

$$= \lim_{t \rightarrow 0} \frac{f(x+t) - f(x)}{-t} = -\lim_{t \rightarrow 0} \frac{f(x+t) - f(x)}{t}$$

$$= -f'(x) \text{ as desired.}$$

Argument D:

Given f is even, so $f(x) = f(-x)$. Take the derivative of both sides: Then $f'(x) = -f'(-x)$ by the chain rule. So f' is odd.

Argument E:

f is even, so $f(x) = f(-x)$.

Multiply both sides by -1 : $-f(x) = -f(-x)$.

Factor in -1 on the left: $f(-x) = -f(-x)$

f is even so we can substitute $f(-x) = f(x)$ on the right:

So $f(-x) = -f(x)$.

Take the derivative of both sides $f'(-x) = -f'(x)$.

so f' is odd, as required.