

Shell Problems [section 6.3 pg 437 # 1-20]

#	region boundary	axis of rev				thickness		radius	height	definite integral
		x-axis	y-axis	x=?	y=?	dx	dy			$2\pi \int_{\text{lower}}^{\text{upper}} \text{radius} \cdot \text{height} \cdot \text{thickness}$
1	$Y=X, Y=0, X=2$	■				✓		X	$Y=X$	$2\pi \int_0^2 X \cdot X dx = 2\pi \int_0^2 X^2 dx$
2	$Y=1-X, Y=0, X=0$	■				✓		X	$Y=1-X$	$2\pi \int_0^1 X(1-X) dx = 2\pi \int_0^1 (X-X^2) dx$
3	$Y=\sqrt{X}, Y=0, X=4$	■				✓		X	$Y=\sqrt{X}$	$2\pi \int_0^4 X\sqrt{X} dx = 2\pi \int_0^4 X^{3/2} dx$
4	$Y=X^2+4, X=0, Y=8$	■				✓		X	$8-Y = 4-X^2$	$2\pi \int_4^8 X(4-X^2) dx = 2\pi \int_4^8 (4X-X^3) dx$
5	$Y=X^2, Y=0, X=2$	■				✓		X	$Y=X^2$	$2\pi \int_0^2 X \cdot X^2 dx = 2\pi \int_0^2 X^3 dx$
6	$Y=\frac{1}{2}X^2, Y=0, X=6$	■				✓		X	$Y=\frac{1}{2}X^2$	$2\pi \int_0^6 X(\frac{1}{2}X^2) dx = \pi \int_0^6 X^3 dx$
7	$Y=X^2, Y=4X-X^2$	■				✓		X	$(4X-X^2) - (X^2)$	$2\pi \int_0^2 X[(4X-X^2)-X^2] dx = 4\pi \int_0^2 (2X^2-X^3) dx$
8	$Y=4-X^2, Y=0$	■				✓		X	$Y=4-X^2$	$2\pi \int_0^2 X(4-X^2) dx = 2\pi \int_0^2 (4X-X^3) dx$
9	$Y=4X-X^2, X=0, Y=4$	■				✓		X	$4-Y = 4-(4X-X^2)$	$2\pi \int_0^2 X[4-(4X-X^2)] dx = 2\pi \int_0^2 X^3 dx$
10	$Y=2X, Y=4, X=0$	■				✓		X	$4-Y = 4-2X$	$2\pi \int_0^2 X(4-2X) dx = 4\pi \int_0^2 (2X-X^2) dx$
11	$Y=\frac{1}{\sqrt{2\pi}} e^{-X^2/2}, Y=0, X=0, X=1$	■				✓		X	$Y=\frac{1}{\sqrt{2\pi}} e^{-X^2/2}$	$2\pi \int_0^1 X \cdot \frac{1}{\sqrt{2\pi}} e^{-X^2/2} dx = \sqrt{2\pi} \int_0^1 X e^{-X^2/2} dx$
12	$Y=\begin{cases} \frac{\sin X}{X}, & X>0 \\ 1, & X=0 \end{cases}, Y=0, X=0, X=\pi$	■				✓		X	$Y=\frac{\sin X}{X}$	$2\pi \int_0^\pi X \cdot \frac{\sin X}{X} dx = 2\pi \int_0^\pi \sin X dx$

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		x-axis	y-axis	x=?	y=?	dx	dy			
13	$y=x, y=0, x=2$	■					✓	y	$2-x$ $=2-y$	$2\pi \int_0^2 y(2-y)dy = 2\pi \int_0^2 (2y-y^2)dy$
14	$y=2-x, y=0, x=4$	■					✓	y	$4-x$ $=4-(2-y)$	$2\pi \int_{-2}^0 y(4-(2-y))dy = 2\pi \int_{-2}^0 (2y+y^2)dy$
15	$y=\frac{1}{x}, x=1, x=2, y=0$	■					✓	y	1 for $y < \frac{1}{2}$ $\frac{1}{y} - 1$ for $y > \frac{1}{2}$	$2\pi \int_0^{\frac{1}{2}} y \cdot 1 dy + 2\pi \int_{\frac{1}{2}}^1 y(\frac{1}{y}-1)dy = 2\pi \left(\int_0^{\frac{1}{2}} y dy + \int_{\frac{1}{2}}^1 (1-y) dy \right)$
16	$x+y^2=16, x=0$	■					✓	y	$x=16-y^2$	$2\pi \int_0^4 y(16-y^2)dy = 2\pi \int_0^4 (16y-y^3)dy$
17	$y=x^2, y=4x-x^2$			4			✓	$4-x$	$(4x-x^2)$ $-x^2$	$2\pi \int_0^2 (4-x)[(4x-x^2)-x^2]dx = 4\pi \int_0^2 (x^3-6x^2+8x)dx$
18	same as 17			2			✓	$2-x$	$(4x-x^2)$ $-x^2$	$2\pi \int_0^2 (2-x)[(4x-x^2)-x^2]dx = 4\pi \int_0^2 (x^3-4x^2+4x)dx$
19	$y=4x-x^2, y=0$			5			✓	$5-x$	$y=4x-x^2$	$2\pi \int_0^4 (5-x)(4x-x^2)dx = 2\pi \int_0^4 (x^3-9x^2+20x)dx$
20	$y=\sqrt{x}, y=0, x=4$			6			✓	$6-x$	$y=\sqrt{x}$	$2\pi \int_0^4 (6-x)\sqrt{x}dx = 2\pi \int_0^4 (6x^{\frac{1}{2}}-x^{\frac{3}{2}})dx$